EM-Tutorübung, Blatt 6 – 7.6.2010

Einführung

Gaußsches Gesetz: → differentielle Form

$$\operatorname{div} \vec{D} = \rho$$

→ integrale Form

$$\iint_{\partial V} \vec{D}\, d\vec{a} = Q$$

Satz v. Gauß:

$$\iint_{\partial V} \vec{u}\, d\vec{a} = \iiint_{V} \operatorname{div} \vec{u}\, dV$$

$$Q = \iint_{\partial V} \vec{D}\, d\vec{a} = \iiint_{V} \operatorname{div} \vec{D}\, dV$$

$$Q = \iiint_{V} \rho\, dV$$

$$\Downarrow \quad \operatorname{div} \vec{D} = \rho$$

1. Maxwellsche Gleichung

Aufgabe 14:

geg: $\rho(r) = \rho_0 \left(1 - \left(\frac{r}{r_0}\right)^4\right)$

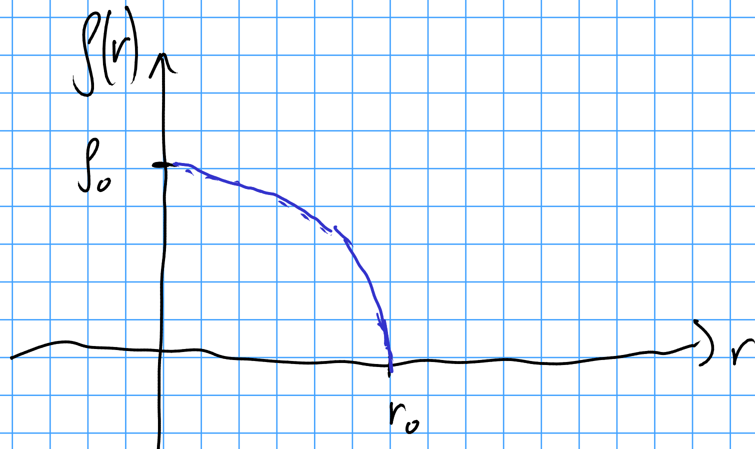
ges: Skizze

Lös: Berechnung wichtiger Punkte:

$$\rho(r=0) = \rho_0$$

$$\rho(r) \overset{!}{=} 0 \iff r = \pm r_0$$

$\Rightarrow$ achsensymmetrisch



b) $$\rho(r) = \rho_0 \left(1 - \left(\frac{r}{r_0}\right)^4\right)$$

Fall 1: $r < r_0$ $$Q = \int_0^H \int_0^{2\pi} \int_0^r \rho_0 \left(1 - \left(\frac{r'}{r_0}\right)^4\right) \underbrace{r' dr' d\varphi dz}_{dV} =$$

$$= 2\pi H \rho_0 \int_0^r r' - \frac{r'^5}{r_0^4} dr' = 2\pi H \rho_0 \left[\frac{1}{2} r'^2 - \frac{1}{6} \frac{r'^6}{r_0^4}\right]_0^r =$$

$$= 2\pi H \rho_0 \left[\frac{1}{2} r^2 - \frac{1}{6} \frac{r^6}{r_0^4}\right]$$

Fall 2: $r > r_0$

$$Q_2(r) = Q_1(r)\big|_{r=r_0} = 2\pi H \rho_0 \left[\frac{1}{2} r_0^2 - \frac{1}{6} r_0^2\right] =$$

$$= \underline{\underline{2\pi H \rho_0 \frac{1}{3} r_0^2}}$$

c) $$E_r(r) = \frac{\rho_0}{\varepsilon}\left(\frac{r}{2} - \frac{r^5}{6 r_0^4}\right) \quad r < r_0$$

$$Q_1(r) = \iint_{\partial V} \vec{D}(\vec{r})\, d\vec{a} = \int_0^H \int_0^{2\pi} D_r(r) \cdot \underbrace{\vec{e}_r \vec{e}_r}_{=1} r \, d\varphi \, dz =$$

$$= D_r(r) \cdot r \cdot 2\pi H$$

$$\Rightarrow D_r(r) = \frac{Q_1(r)}{2\pi H r} = \frac{2\pi H \rho_0 \left[\frac{1}{2} r^2 - \frac{1}{6}\frac{r^6}{r_0^4}\right]}{2\pi H r} = \rho_0 \left[\frac{1}{2} r - \frac{1}{6}\frac{r^5}{r_0^4}\right]$$

$$\vec{D} = \varepsilon \vec{E} \Rightarrow \vec{E} = \frac{\vec{D}}{\varepsilon} = \frac{\rho_0}{\varepsilon}\left[\frac{1}{2} r - \frac{1}{6}\frac{r^5}{r_0^4}\right]$$

$dr\vec{e}_r$ $\qquad \vec{E} = E_r(r) \cdot \vec{e}_r$

$$W_{mech} = -W_{el} = -\int_{P_0}^{P_1} \vec{F}_{el}\, d\vec{r} = \quad ; \quad \vec{E} = \frac{\vec{F}_{el}}{Q}$$

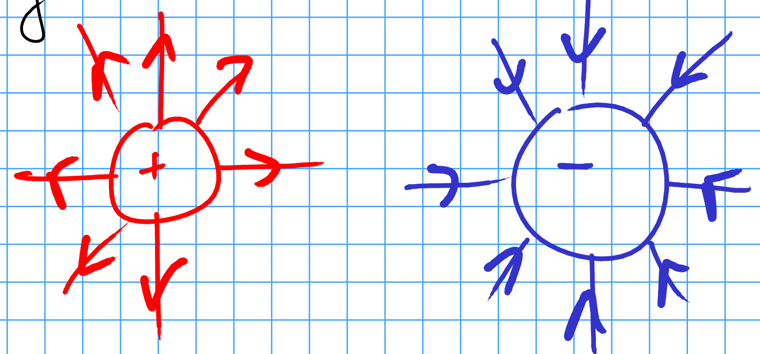
$$= -\int_{r=r_0}^{r=0} Q_P \cdot \frac{\rho_0}{\varepsilon}\left[\frac{1}{2} r - \frac{1}{6}\frac{r^5}{r_0^4}\right] dr =$$

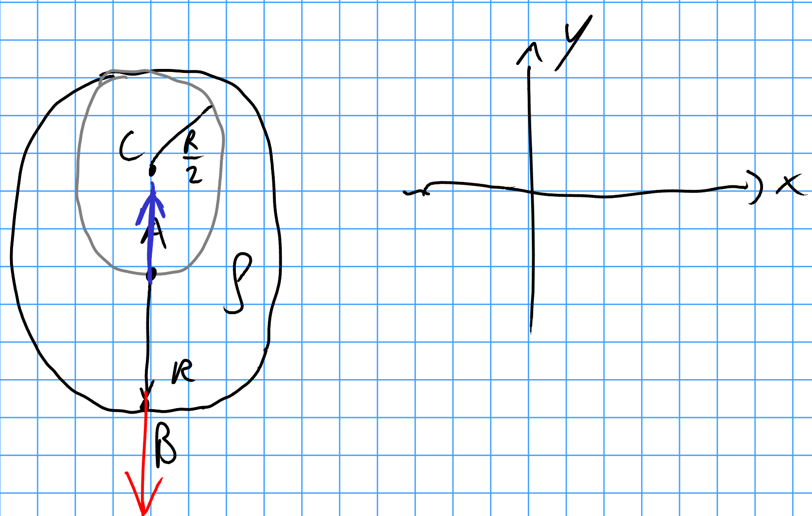
$$= -Q_p \cdot \frac{\rho_0}{\varepsilon}\left[\frac{1}{4}r^2 - \frac{1}{36}\frac{r^6}{r_0^4}\right]_{r=r_0}^{r=0} =$$

$$= +Q_p \cdot \frac{\rho_0}{\varepsilon}\left[\frac{1}{4}r_0^2 - \frac{1}{36}r_0^2\right] = +\frac{2}{9}\,\frac{\rho_0 r_0^2 Q_p}{\varepsilon_0}$$

Aufgabe 15

Vorüberlegung: Feldlinien-Bilder





Berechnung für Punkt A:

$$\vec{E}(A) = \vec{E}_1 + \vec{E}_2$$

$$Q_{ges} = \iiint \rho \, dV = \rho \, \frac{4}{3}\pi R^3$$

$$Q_1 = \iiint \rho \, dV = \rho \, \frac{4}{3}\pi \left(\frac{R}{2}\right)^3 = \frac{\pi}{6}\rho R^3$$

$$\vec{E}(A) = \vec{E}_1 + \vec{E}_2 = \frac{\pi 16 \rho R^3}{4\pi \varepsilon_0 \left(\frac{R}{2}\right)^3} \left(\frac{R}{2}\right) \vec{e}_y + 0$$

$$= \frac{\rho R}{6 \varepsilon} \vec{e}_y$$

$$\vec{E}(B) = \vec{E}_1 + \vec{E}_2 = \frac{\pi 16 \rho R^3}{4\pi \varepsilon_0 \left(\frac{3}{2}R\right)^3} \cdot \left(\frac{3}{2}R\right) \vec{e}_y + \frac{\frac{4\pi}{3} \rho R^3}{4\pi \varepsilon_0 R^3} \cdot R\left(-\vec{e}_y\right) =$$

$$= -\frac{17}{54} \frac{\rho R}{\varepsilon_0} \vec{e}_y$$