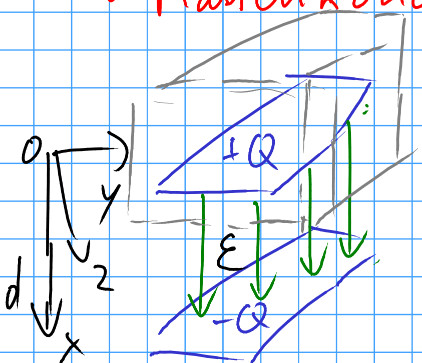
EM-Tutorübung, Blatt 8, 21.6.2010

Kondensatoren / Kondensatoraggregate

$$C = \frac{Q}{U} \qquad Q = \iint_{\partial V} \vec{D}\, d\vec{a}$$

$$U = \int_C \vec{E}\, d\vec{r}$$

- Plattenkondensator




$$Q = \iint_{\partial V} \vec{D}\, d\vec{a} = \int_0^{z_0}\int_0^{y_0} D_x(x)\cdot \vec{e}_x \cdot \vec{e}_x \, dy\, dz$$

$$= D_x(x)\cdot y_0 z_0 = D_x(x)\cdot A$$

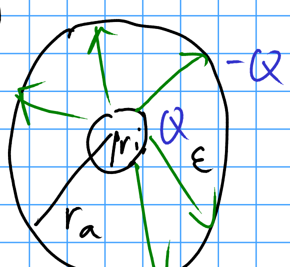
$$\Rightarrow D_x(x) = \frac{Q}{A} \qquad E_x(x) = \frac{Q}{A\varepsilon} = E_x$$

$$U = \int \vec{E}\, d\vec{r} = \int_0^d \frac{Q}{A\varepsilon}\, dx = \frac{Q}{A\varepsilon}\, d$$

$$C = \frac{Q}{U} = \frac{Q}{\frac{Q}{A\varepsilon} d} = \varepsilon \cdot \frac{A}{d}$$

$\Rightarrow$ darf nur geometriebedingte Größen enthalten

Kugelkondensator:




$$\vec{D}(\vec{r}) = D_r(r)\cdot \vec{e}_r$$

$$Q = \iint \vec{D}\, d\vec{a} = \int_0^{\pi}\int_0^{2\pi} D_r(r)\cdot \vec{e}_r \cdot \vec{e}_r\, r^2 \sin\vartheta\, d\varphi\, d\vartheta$$

$$= D_r(r)\, r^2\, 4\pi$$

$$\Rightarrow D_r(r) = \frac{Q}{4\pi r^2}$$

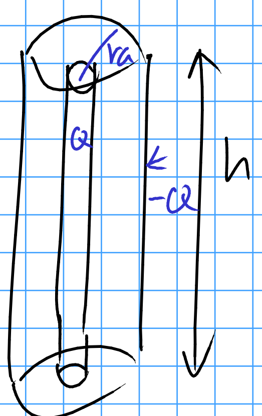
$$\Rightarrow E_r(r) = \frac{Q}{4\pi\varepsilon r^2}$$

$$U = \int \vec{E}\, d\vec{r} = \int_{r_i}^{r_a} \frac{Q}{4\pi\varepsilon r^2}\, dr = \frac{Q}{4\pi\varepsilon}\left[-\frac{1}{r}\right]_{r_i}^{r_a} = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r_i} - \frac{1}{r_a}\right]$$

$$= \frac{Q}{4\pi\varepsilon}\,\frac{r_a - r_i}{r_a r_i}$$

$$\Rightarrow C = \frac{Q}{U} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{4\pi\varepsilon}\,\frac{r_a - r_i}{r_a r_i}} = \underline{4\pi\varepsilon\,\frac{r_i r_a}{r_a - r_i}}$$

Zylinderkondensator




$$Q = \iint \vec{D}\, d\vec{a} = \int_0^h \int_0^{2\pi} D_r(r)\, \vec{e}_r\, \vec{e}_r\, r\, d\varphi\, dz$$

$$= D_r(r)\, 2\pi r h$$

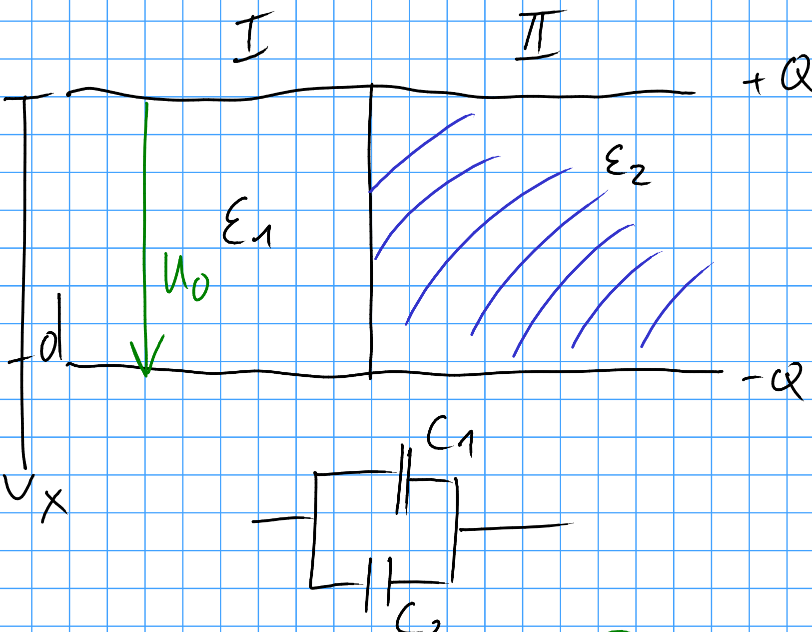
$$\Rightarrow D_r(r) = \frac{Q}{2\pi r h}, \quad E_r(r) = \frac{Q}{2\pi r h \varepsilon}$$

$$U = \int \vec{E}\, d\vec{r} = \int_{r_i}^{r_a} \frac{Q}{2\pi\varepsilon r h}\, dr = \frac{Q}{2\pi\varepsilon h}\Big[\ln r\Big]_{r_i}^{r_a}$$

$$= \frac{Q}{2\pi\varepsilon h} \ln \frac{r_a}{r_i}$$

$$\Rightarrow C = \frac{Q}{U} = 2\pi\varepsilon h \cdot \frac{1}{\ln \frac{r_a}{r_i}}$$

# Kondensator-Aggregate




$U = U_0$ für Bereich I, II

nach $U = \int E\,d\vec{r}$ muss

also auch das <u>E-Feld in beiden bereichen konstant sein</u>

$$E_I = E_{II}$$

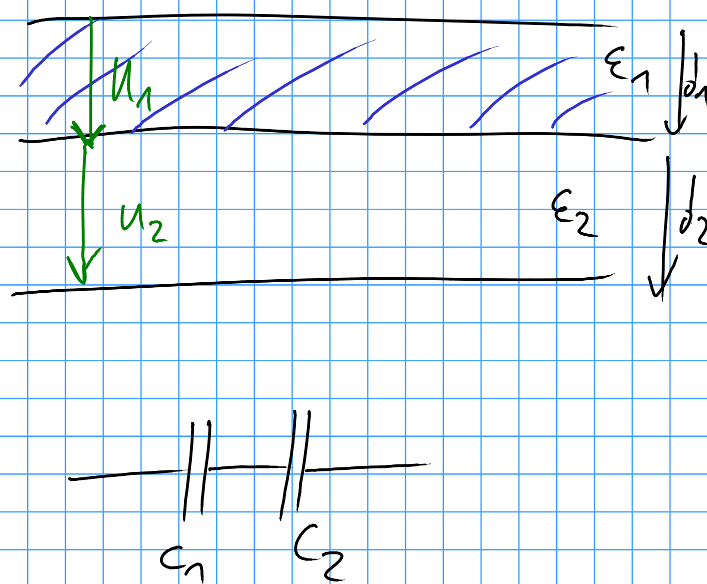
$$Q = \iint_{\partial V_1} \vec{D}_1\, d\vec{a} + \iint_{\partial V_2} \vec{D}_2\, d\vec{a} =$$

$$= D_1 \cdot A_1 + D_2 \cdot A_2 =$$

$$= \varepsilon_1 E \cdot A_1 + \varepsilon_2 E \cdot A_2$$

$$= E\,(\varepsilon_1 A_1 + \varepsilon_2 A_2)$$

$$\Rightarrow E = \frac{Q}{\varepsilon_1 A_1 + \varepsilon_2 A_2}$$




$\Rightarrow$ E-Feld kann nicht konstant sein

$\Rightarrow$ D ist hier gleich

$$Q = \iint \vec{D}\, d\vec{a} = D \cdot A = \begin{cases} \varepsilon_1 E_1 \cdot A_1 \\ \varepsilon_2 \cdot E_2 \cdot A_2 \end{cases}$$

$$U = \int_0^{d_1} E_1\, dr + \int_{d_1}^{d_1+d_2} E_2\, dr =$$

$$= \frac{Q}{\varepsilon_1 A_1} d_1 + \frac{Q}{\varepsilon_2 A_2} d_2$$

$$C_{ges} = \frac{Q}{U} = \frac{1}{\underbrace{\frac{d_1}{\varepsilon_1 A_1}}_{C_1} + \underbrace{\frac{d_2}{\varepsilon_2 A_2}}_{C_2}}$$

$$\frac{1}{C_{ges}} = \frac{1}{C_1} + \frac{1}{C_2}$$

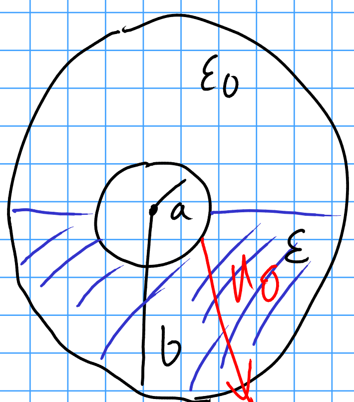
$$U = \int \vec{E}\, d\vec{r} = E \cdot d$$

$$\Rightarrow C_{ges} = \frac{Q}{U} = \frac{Q}{\frac{Q}{\varepsilon_1 A_1 + \varepsilon_2 A_2} d} =$$

$$= \underbrace{\varepsilon_1 \frac{A_1}{d}}_{C_1} + \underbrace{\varepsilon_2 \frac{A_2}{d}}_{C_2} = C_1 + C_2$$

## Aufgabe 19

a) s.o. (Einführung)

b) 


$\hat{=}$ Parallelschaltung

$$Q = \iint_{\partial V_1} D_1\, d\vec{a} + \iint_{\partial V_2} D_2\, d\vec{a} =$$

$$= \int_0^{\pi/2}\int_0^{2\pi} D_1(r)\, r^2 \sin\vartheta\, d\varphi\, d\vartheta + \int_{\frac{\pi}{2}}^{\pi}\int_0^{2\pi} D_2\, r^2 \sin\vartheta\, d\varphi\, d\vartheta$$

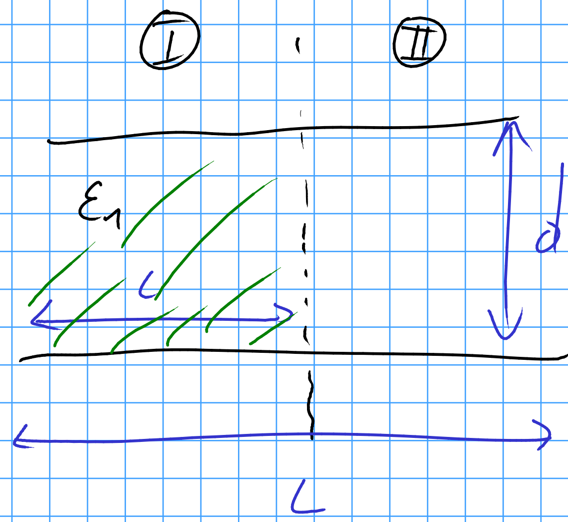
$$= \varepsilon_0 E_r(r) \cdot 2\pi r^2 + \varepsilon E_r(r) \cdot 2\pi r^2$$

$$= E_r(r)\, 2\pi r^2 (\varepsilon_0 + \varepsilon)$$

$$U = \int \vec{E}\, d\vec{r} = \frac{Q}{2\pi(\varepsilon_0 + \varepsilon)} \left(\frac{1}{r_i} - \frac{1}{r_a}\right)$$

$$\Rightarrow C = \frac{Q}{U} = 2\pi(\varepsilon_0 + \varepsilon)\frac{r_a r_i}{r_a - r_i}$$

## Aufgabe 20




a) $C_0 = \varepsilon_0 \cdot \frac{A}{d}$

b) $\Rightarrow C_0 = \frac{Q_0}{U_0} \Rightarrow U_0 = \frac{Q_0}{C_0}$

$\Rightarrow E_0 = \frac{U_0}{d} = \frac{Q_0}{d\,C_0}$

$\Rightarrow \sigma_0 = \frac{Q_0}{A}$

c) $E_1 = E_2$ (da Parallelschaltung zweier Kondensatoren, vgl. oben)

$$\sigma_0 = \sigma_1 + \sigma_2$$

$$Q_0 = \iint_{\partial V_1} \vec{D}_1\, d\vec{a} + \iint_{\partial V_2} \vec{D}_2\, d\vec{a} = \underbrace{|D_1|}_{\sigma_1} \cdot A_1 + \underbrace{|D_2|}_{\sigma_2} \cdot A_2$$

$$D_1 = \overbrace{\varepsilon_1}^{\varepsilon_r \varepsilon_0} \cdot E = \sigma_1$$

$$D_2 = \varepsilon_0 \cdot E = \sigma_2$$

$$\sigma_1 = \varepsilon_r \sigma_2$$

b) $$Q_0 = \sigma_1 \cdot A_1 + \sigma_2 A_2 =$$

$$= \varepsilon_r \sigma_2 A_1 + \sigma_2 A_2 =$$

$$= \varepsilon_r \sigma_2 \, x \cdot A + \sigma_2 (1-x) A$$

$$= \sigma_2 A \left[ \varepsilon_r x + 1 - x \right] \Rightarrow \sigma_2 = \frac{Q}{A(\varepsilon_r x + 1 - x)} = \frac{\sigma_0}{\varepsilon_r x + 1 - x}$$

c) $$C_1 = \frac{Q_1}{U} = \frac{\sigma_1 \cdot A_1}{U} = \frac{\sigma_1 \cdot x \cdot A}{U} = \frac{\varepsilon_0 \varepsilon_r \cdot \cancel{U} \cdot x \cdot A}{\cancel{U} \cdot d} =$$

$$E = \frac{U}{d}$$

$$= \varepsilon_0 \varepsilon_r \cdot x \cdot \frac{A}{d} = \underline{\underline{\varepsilon_r \cdot x \cdot C_0}}$$

$$\frac{D_1}{\varepsilon_0 \varepsilon_r} = \frac{U}{d} \Leftrightarrow \frac{\sigma_1}{\varepsilon_0 \varepsilon_r} = \frac{U}{d}$$

$$C_2 = \frac{Q_2}{U} = \frac{\sigma_2 \cdot A_2}{U} = \frac{\varepsilon_0 \frac{\cancel{U}}{d} (1-x) A}{\cancel{U}} = \varepsilon_0 \frac{A}{d} (1-x)$$

$$= C_0 (1-x)$$

$$\frac{D_2}{\varepsilon} = \frac{U}{d} \Leftrightarrow \frac{\sigma_2}{\varepsilon_0} = \frac{U}{d}$$

d) $$C(x) = C_1(x) + C_2(x) = \varepsilon_r x C_0 + C_0 (1-x) =$$

$$= C_0 \left[ \varepsilon_r x + 1 - x \right]$$

e) $C(x) = \frac{Q_0}{U_{Q_0}} \Rightarrow U_{Q_0} = \frac{Q_0}{C(x)}$

f) $W_{e,Q_0}(x) = \frac{1}{2} C(x) U^2 = \frac{1}{2} C(x) U_{Q_0}^2 = \frac{1}{2} C(x) \cdot \frac{Q_0^2}{C(x)^2}$

$= \frac{1}{2} \frac{Q_0^2}{C(x)}$

g) $C(x) = \frac{Q}{U}\Big|_{U=U_B} \Rightarrow Q = C(x) \cdot U_B$

h) $W_{e,B}(x) = \frac{1}{2} C(x) \cdot U_B^2$