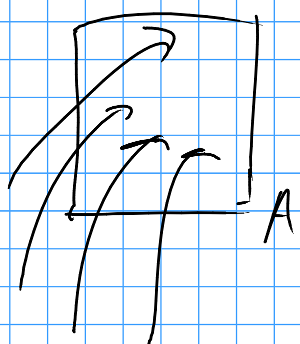
EM Tutorübung, Blatt 9 - 28.06.2010

Ergänzung:

$$w_{el} = \frac{1}{2}\vec{E}\cdot\vec{D} = \frac{1}{2}\varepsilon|\vec{E}|^2 = \frac{1}{2}\frac{|\vec{D}|^2}{\varepsilon}$$

stationäre Ströme (stationäre: $\frac{d}{dt}x(t) = 0$)

$$I = \iint_A \vec{j}\, d\vec{a}$$



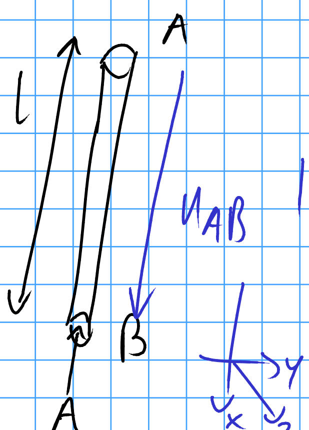
$$\vec{j} = q\cdot n\cdot\vec{v} = \sigma\cdot\vec{E} = \underbrace{q\cdot n\cdot\mu}_{\sigma}\cdot\vec{E}$$

$$\vec{v} = \mu\vec{E}$$

$$\boxed{\vec{j} = \sigma\cdot\vec{E}}$$ ohmsches Gesetz in differentieller Form

$\sigma = 1\,\frac{S}{m}$ Leitfähigkeit

→ ohmsches Gesetz in integraler Form



Annahme: - homogener Querschnitt
- homogene Leitfähigkeit
- Länge l

$$|\vec{E}| = \frac{U_{AB}}{l}$$

$$\vec{j} = \sigma\cdot\vec{E} \Rightarrow R = \frac{U}{I}$$

(Leiter-querschnittsfläche)

$$I = \iint_A \vec{j}\, d\vec{a} = \iint_A \sigma\cdot\vec{E}\, d\vec{a} = \iint_A \sigma\cdot\underbrace{\frac{U_{AB}}{L}\cdot\vec{e}_x}_{\vec{E}}\,\vec{e}_x\, dy\, dz =$$

$$= \sigma \cdot \frac{U_{AB}}{l} \cdot A$$

$$\Rightarrow \frac{U_{AB}}{I} = \frac{1}{\sigma} \cdot \frac{l}{A} = \rho \cdot \frac{l}{A} = R$$

ρ: spez. Widerstand

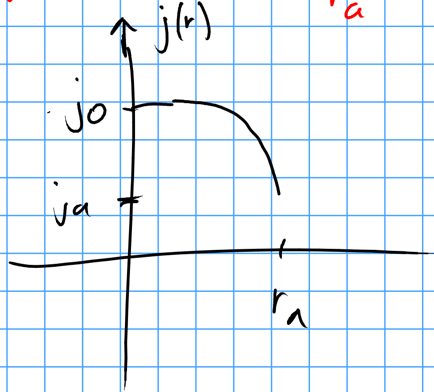
Aufgabe 21

<u>Bekannt:</u> $j(r) = ar^2 + br + c$; $j'(r) = 2ar + b$

$j(0) = j_0 \Rightarrow c = j_0 \quad (I)$

$j(r_a) = j_a \Rightarrow ar_a^2 + br_a + j_0 = j_a \Rightarrow a = \frac{j_a - j_0}{r_a^2} \quad (III)$

$j'(0) = 0 \Rightarrow b = 0 \quad (II)$



$$\Rightarrow j(r) = \frac{j_a - j_0}{r_a^2} r^2 + j_0$$

b) $$I = \iint_A \vec{j}\, d\vec{a} = \int_0^{2\pi}\int_0^{r_a} \left(\frac{j_a - j_0}{r_a^2} r^2 + j_0\right) \vec{e}_z \cdot \vec{e}_z \, r\, dr\, d\varphi =$$

$$= 2\pi \int_0^{r_a} \frac{j_a - j_0}{r_a^2} r^3 + j_0 r \, dr = 2\pi \left[\frac{j_a - j_0}{4 r_a^2} r^4 + \frac{1}{2} j_0 r^2\right]_0^{r_a} =$$

$$= 2\pi \left(\frac{j_a - j_0}{4 r_a^2} r_a^4 + \frac{1}{2} j_0 r_a^2\right) = \pi r_a^2 \frac{j_a + j_0}{2}$$

c) $j(r) = j_0$

$$I = \iint_A \vec{j}\, d\vec{a} = j_0 r_a^2 \pi$$

Aufgabe 22

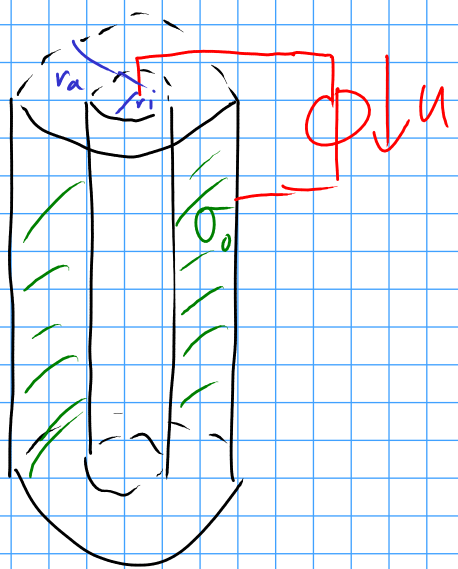
$$\vec{j} = j_0\, e^{-r/R_0}\, \vec{e}_\varphi$$

$$I = \iint_A \vec{j}\, d\vec{a} = \int_{r_1}^{r_2}\int_0^{z_0} j_0 e^{-r/R_0}\, \vec{e}_\varphi \vec{e}_\varphi\, dz\, dr =$$

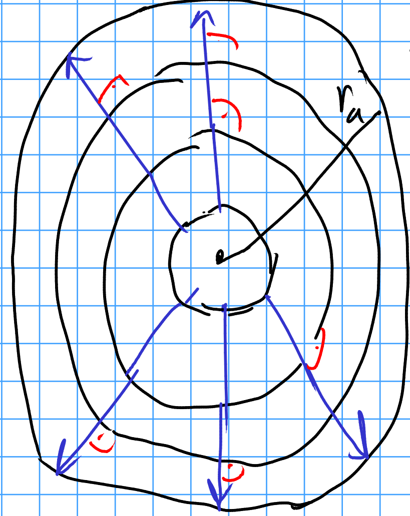
$$= j_0 z_0 \int_{r_1}^{r_2} e^{-r/R_0}\, dr = j_0 z_0 \left[-R_0 e^{-r/R_0}\right]_{r_1}^{r_2} =$$

$$= j_0 z_0 \left[-R_0 e^{-r_2/R_0} + R_0 e^{-r_1/R_0}\right]$$

23. Aufgabe



a) Draufsicht



E-Feld

konzentrische Kreise: Äquipotentiallinien

b) $\vec{j} = \sigma \cdot \vec{E}$

$\Rightarrow \vec{E}$ in $\vec{e}_r$-Richtung

$\vec{E} = E(r)\,\vec{e}_r$

c) $$I = \iint_A \vec{j}\, d\vec{a} = \int_0^L \int_0^{2\pi} j(r)\, \vec{e}_r \vec{e}_r\, r\, d\varphi\, dz =$$

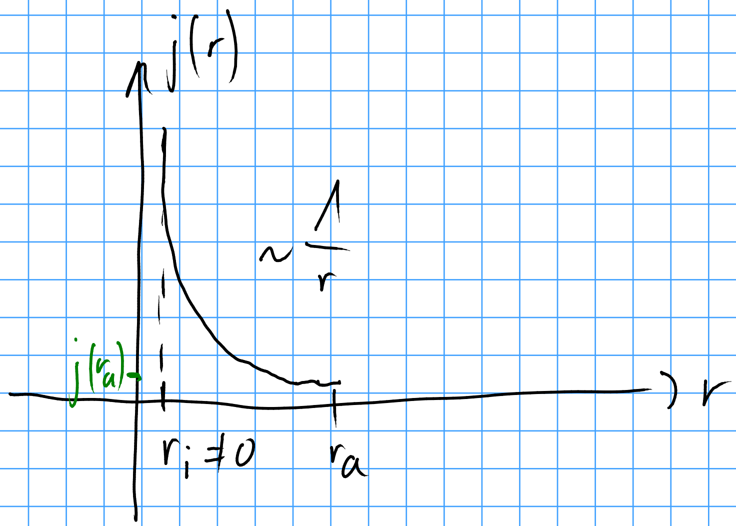
$$= j(r) \cdot r \cdot 2\pi \cdot L \quad \Rightarrow j(r) = \frac{1}{2\pi r L}$$

$$\vec{j} = \sigma \cdot \vec{E} \quad \Rightarrow \quad \vec{E} = \frac{\vec{j}}{\sigma} = \frac{1}{2\pi r L \sigma}$$

$$\Phi = -\int_{r_a}^{r} \vec{E}\, d\vec{r} = -\int_{r_a}^{r} \frac{1}{2\pi r' L \sigma}\, dr' = -\frac{1}{2\pi L \sigma} \int_{r_a}^{r} \frac{1}{r'}\, dr'$$

$$= -\frac{1}{2\pi L \sigma} \left(\ln \frac{r}{r_a} \right) = \frac{1}{2\pi L \sigma} \ln \frac{r_a}{r}$$

d) $j(r_a) = \frac{1}{2\pi L r_a}$ $\qquad$ $j(r_i) = \frac{1}{2\pi L r_i}$



e) $U_{AB} = \phi_1 - \phi_2 = \frac{1}{2\pi\sigma L}\left(\ln \frac{r_a}{r_i}\right)$

$$\vec{E} = \frac{\overset{?}{j}(r)}{\sigma} = \frac{1}{2\pi L r \sigma} = \frac{U_{AB} \cdot 2\pi \cancel{\sigma} \cancel{L}}{2\pi \cancel{L} r \cancel{\sigma} \cdot \ln\left(\frac{r_a}{r_i}\right)} = \frac{U_{AB}}{\ln\left(\frac{r_a}{r_i}\right)} \cdot \frac{1}{r}$$