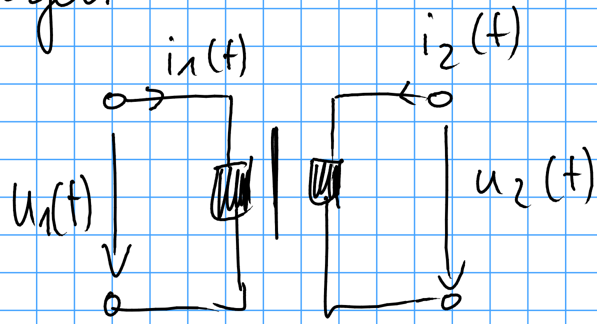
EMF-Tutorübung, Blatt 10 21.01.2011

Wiederholung:

Transformatorgleichungen



$$u_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$$

M: Koppelinduktivität

$$u_2(t) = M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt}$$

$$k = \frac{M}{\sqrt{L_1 \cdot L_2}}$$

($k=1$: idealer Übertrager)

Aufgabe 20

a) $M = k \cdot \sqrt{L_1 \cdot L_2} = 0{,}8 \sqrt{0{,}1\,H \cdot 10\,H} = 0{,}8\,H$

b) $I_2 = 0$ $\Leftrightarrow$ sekundärseitiger Leerlauf

$$u_1(t) = L_1 \frac{di_1(t)}{dt} + \underbrace{M \cdot \frac{di_2(t)}{dt}}_{=0}$$

und $i_1(t) = I_0 \left(1 - e^{-\alpha t}\right)$

$$u_1(t) = L_1 \frac{d}{dt}\left(I_0\left(1-e^{-\alpha t}\right)\right) = L_1 I_0 \alpha e^{-\alpha t}$$

c) ges: $u_2(t)$

Lös: $$u_2(t) = M \frac{di_1(t)}{dt} + \underbrace{L_2 \frac{di_2(t)}{dt}}_{\equiv 0}$$

$$= M I_0 \alpha e^{-\alpha t}$$

d) $$W_m = \frac{1}{2} L_1 I_1^2 + \underbrace{\frac{1}{2} M I_1 \cdot I_2}_{\equiv 0} + \underbrace{\frac{1}{2} L_2 I_2^2}_{\equiv 0}$$

$$= \frac{1}{2} L_1 \left(I_0^2 \left(1-e^{-\alpha t}\right)^2\right)$$

$$= \frac{1}{2} L_1 I_0^2 \left(1-e^{-\alpha t}\right)^2$$

1. Fall $t = \frac{1}{\alpha}$: $W_m = \frac{1}{2} L_1 I_0^2 \left(1-e^{-1}\right)^2$

2. Fall $t \to \infty$: $\lim_{t \to \infty} W_m = \frac{1}{2} L_1 I_0^2$

e) $U_2(t) = 0$

$$\begin{cases} u_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} \\ 0 = M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt} \end{cases}$$

$$\frac{di_2(t)}{dt} = -\frac{M}{L_2}\frac{di_1(t)}{dt}$$

$$\Rightarrow u_1(t) = L_1 \frac{di_1(t)}{dt} - \frac{M^2}{L_2}\frac{di_1(t)}{dt} =$$

$$= \frac{di_1(t)}{dt}\left(L_1 - \frac{M^2}{L_2}\right) =$$

$$= I_0 \alpha e^{-\alpha t}\left(L_1 - \frac{M^2}{L_2}\right)$$

$$k = \frac{M}{\sqrt{L_1 L_2}} \Rightarrow M^2 = k^2 L_1 \cdot L_2$$

$$= I_0 \alpha e^{-\alpha t}\left(L_1 - \frac{k^2 L_1 \cancel{L_2}}{\cancel{L_2}}\right) = \underline{I_0 \alpha e^{-\alpha t} L_1 (1-k^2)}$$

Aufgabe 21

a) $M = k\sqrt{L_1 L_2}$

b) sekundärseitiger Leerlauf $I_2 = 0$

$$u_1(t) = u_0(1-e^{-\alpha t})$$

$$u_1(t) = L_1 \frac{di_1(t)}{dt} + \underbrace{M\frac{di_2(t)}{dt}}_{=0}$$

$$\frac{di_1(t)}{dt} = \frac{1}{L_1} u_1(t)$$

$$\Rightarrow i_1(t) = \int \frac{1}{L_1} u_1(t')\,dt' = \frac{1}{L_1}\int u_0(1-e^{-\alpha t'})\,dt' =$$

$$= \frac{u_0}{L_1} \int 1 - e^{-\alpha t'}\, dt' = \frac{u_0}{L_1} \left[t' + \frac{1}{\alpha} e^{-\alpha t'} \right] + C$$

$$= \frac{u_0}{L_1} \left[t + \frac{1}{\alpha} e^{-\alpha t} \right] + C$$

Randbedingung: $i_1(t=0) = 0$

$$\frac{u_0}{L_1 \alpha} + C = 0$$

$$C = -\frac{u_0}{L_1 \alpha}$$

$$\Rightarrow i_1(t) = \frac{u_0}{L_1} \left(t - \frac{1}{\alpha} + \frac{1}{\alpha} e^{-\alpha t} \right)$$

Exkurs:

$$x'(t) = f(t)\, g(x)$$

$$x(t_0) = x_0$$

Ansatz durch Seperation:

$$\int_{x_0}^{x} \frac{1}{g(\eta)}\, d\eta = \int_{t_0}^{t} f(\xi)\, d\xi$$

$$\frac{d}{dt} i_1(t) = \underbrace{\frac{u_0}{L_1} \left(1 - e^{-\alpha t} \right)}_{f(t)} \qquad g(i_1) = 1$$

$i_1 \hat{=} x$

$t \hat{=} t$

$$\int_0^{i_1} \frac{1}{g(\eta)}\, d\eta = \int_0^t f(\xi)\, d\xi$$

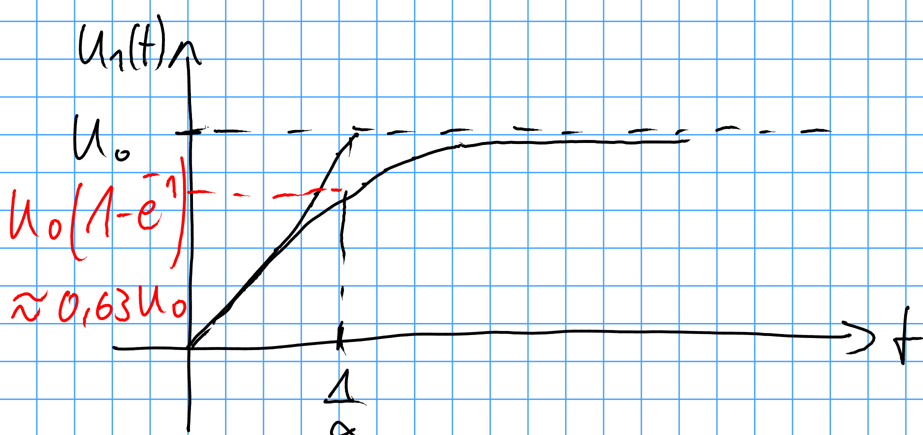
$$\int_0^{i_1} 1\, d\eta = \int_0^t \frac{u_0}{L_1}\left(1 - e^{-\alpha\xi}\right) d\xi$$

$$i_1 = \frac{u_0}{L_1}\left[\xi + \frac{1}{\alpha} e^{-\alpha\xi}\right]_0^t =$$

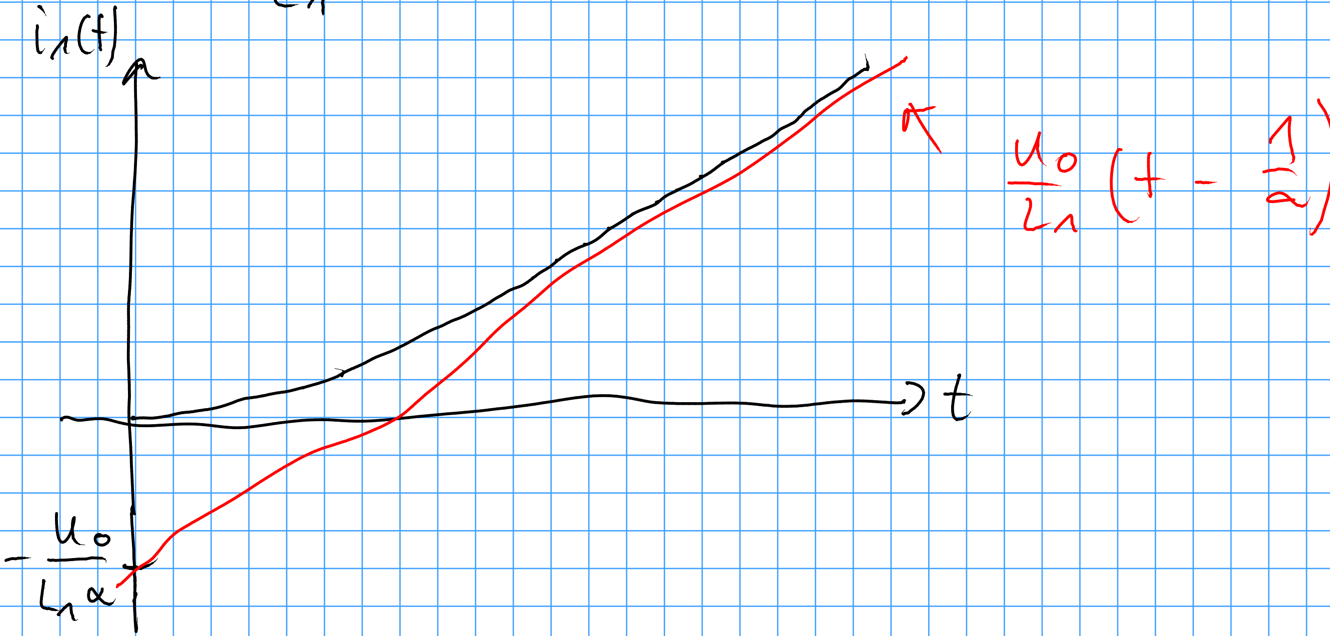
$$= \ldots$$

d) Skizze

$$u_1(t) = u_0\left(1 - e^{-\alpha t}\right)$$



$$i_1(t) = \frac{u_0}{L_1}\left(t + \frac{1}{\alpha}e^{-\alpha t} - \frac{1}{\alpha}\right)$$



$$u_2(t) = M \cdot \frac{di_1(t)}{dt} + \underbrace{L_2 \frac{di_2(t)}{dt}}_{\equiv 0} =$$

$$= \frac{u_0 M}{L_1}\left(1 - e^{-\alpha t}\right)$$

e) $u_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$

$u_2(t) = M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt} = -L_3 \frac{di_2(t)}{dt}$

f) ges: $i_1(t)$, $i_2(t)$

Lös: $\frac{di_2(t)}{dt} = -\frac{u_2(t)}{L_3}$

$$\left[u_1(t) = L_1 \frac{di_1(t)}{dt} - \frac{M}{L_3} u_2(t) \right]$$

$$\dot{i}_1 = \frac{-L_2 \dot{i}_2 - L_3 \dot{i}_2}{M} =$$

$$= \frac{-\dot{i}_2}{M} (L_2 + L_3) \quad (*)$$

$$\Rightarrow u_1 = \left(-\frac{L_1}{M} (L_2 + L_3) + M \right) \dot{i}_2$$

$$\Rightarrow i_2 = \frac{1}{-\frac{L_1}{M}(L_2 + L_3) + M} \int_0^t u_1(t')\,dt'$$

$\Rightarrow$ i_1 durch einsetzen in $(*)$