EMF-Tutorübung, Blatt 11 – 28.1.2011

Einführung / Wiederholung Wechselstromrechnung

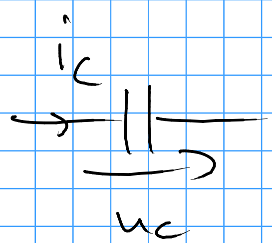
⇒ Annahme: sinusoidale Erregung im eingeschwungenen Zustand:

$$u(t) = \hat{u} \sin(\omega t + \varphi)$$

Zeigerdarstellung: $\underline{\hat{u}} = \hat{u} e^{j\varphi}$

$$u(t) = \mathrm{Im}\left(\underline{\hat{u}}\, e^{j\omega t}\right) =$$

$$= \mathrm{Im}\left(\hat{u}\, e^{j\varphi} \cdot e^{j\omega t}\right)$$



$$i_C = C \dot{u}_C$$

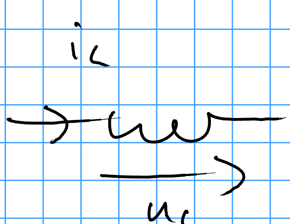
$$I_C(s) = C s U_C(s)$$

$s = \sigma + j\omega$

$\hookrightarrow s = j\omega$

$$I_C(j\omega) = j\omega C U_C(j\omega)$$

$$j\omega C = \frac{I_C(j\omega)}{U_C(j\omega)} = Y$$



$$u_L = L \dot{i}_L$$

$$U_L(s) = s L I_L(s)$$

$$U_L(j\omega) = j\omega L I_L(j\omega)$$

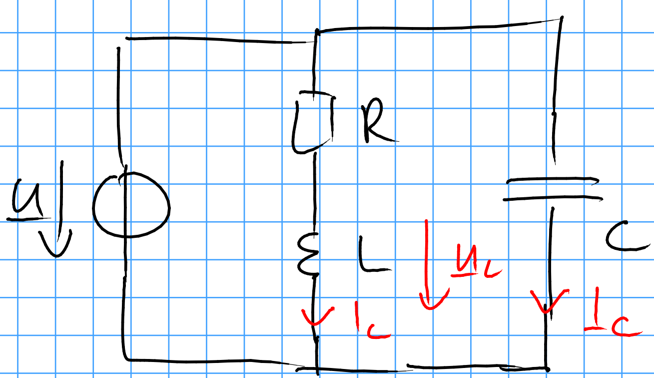
$$\Rightarrow j\omega L = \frac{U_L(j\omega)}{I_L(j\omega)} = Z$$

- Effektivwert: $x(t) = \sqrt{\frac{1}{T}\int_0^T x(t)^2 \, dt}$

Annahme: $x(t) = A\sin(\omega t + \varphi)$

$$X_{eff} = \frac{A}{\sqrt{2}}$$

Aufgabe 22



a) $\underline{Y} = Re(\underline{Y}) + j\,Im(\underline{Y})$

$$= j\omega C + \frac{1}{R + j\omega L} =$$

$$= j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2} =$$

$$= \frac{R}{R^2 + \omega^2 L^2} + j\left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right)$$

b) Phasengleichheit: $Im(\underline{Y}) = 0$

$\hat{\underline{U}} = \hat{U} e^{j\varphi_u}$ $\quad \hat{\underline{I}} = \hat{I} e^{j\varphi_i}$ $\quad \Leftrightarrow \quad \varphi_u = \varphi_i$

$$\underline{Y} = \frac{\hat{I}}{\hat{\underline{U}}} = \frac{\hat{I}\, e^{j\varphi_i}}{\hat{U}\, e^{j\varphi_u}} = \frac{\hat{I}}{\hat{U}} \underbrace{e^{j(\varphi_i - \varphi_u)}}_{\varphi_i = \varphi_u} = \frac{\hat{I}}{\hat{U}}$$

$$\omega_R C - \frac{\omega_R L}{R^2 + \omega_R^2 L^2} = 0$$

$$C = \frac{L}{R^2 + \omega_R^2 L^2}$$

$$C R^2 + \omega_R^2 L^2 C = L$$

$$\Rightarrow \quad \omega_R = \sqrt{\frac{L - CR^2}{L^2 C}} = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

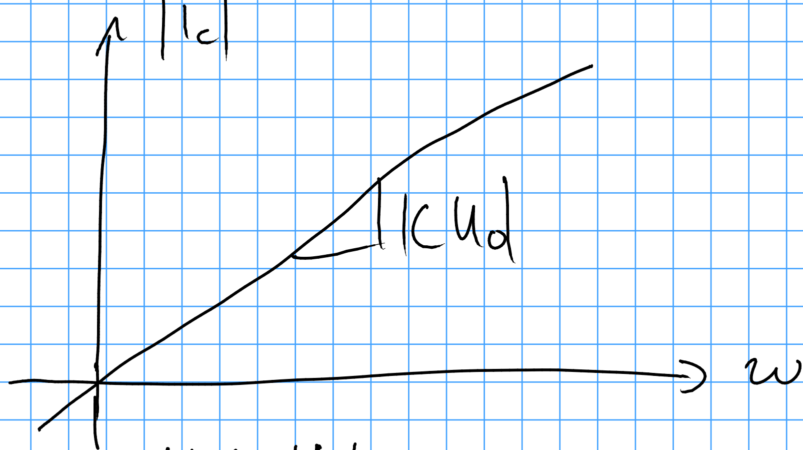
c) ges: $|\underline{I}_R|, |\underline{I}_L|, |\underline{I}_C|$

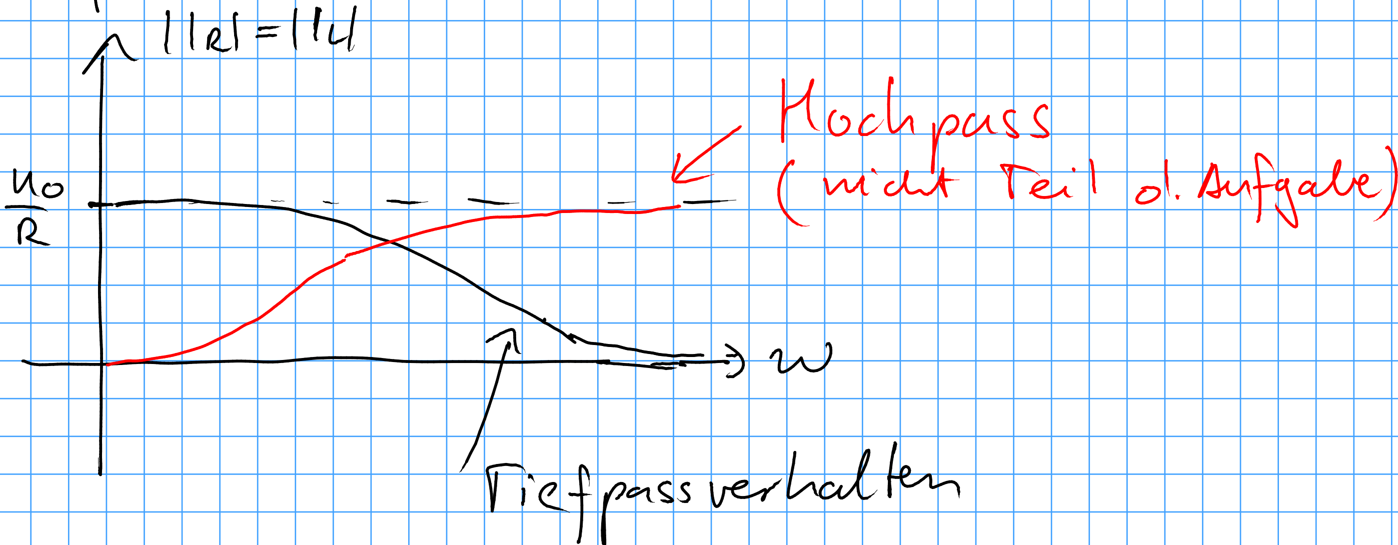
Lös: $|\underline{I}_R| = |\underline{I}_L|$ (Serienschaltung)

$$\Rightarrow |\underline{I}_C| = \frac{|\underline{U}_0|}{\left|\frac{1}{j\omega C}\right|} = |\underline{U}_0| \cdot |\omega C|$$

$$|\underline{U}_L| = |\underline{U}_0| \cdot \frac{|j\omega L|}{|R + j\omega L|} = |\underline{U}_0| \cdot \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}}$$

$$|\underline{I}_L| = \left|\frac{\underline{U}_L}{j\omega L}\right| = \frac{|\underline{U}_0|}{\sqrt{R^2 + \omega^2 L^2}} = |\underline{I}_R|$$





Aufgabe 13

a) $\underline{Z}_1 = R_1 + j\omega L_1 + \dfrac{1}{j\omega C_1} =$

$= R_1 + j\omega L_1 - j\dfrac{1}{\omega C_1} = R_1 + j\left(\omega L_1 - \dfrac{1}{\omega C_1}\right)$

$= R_1 + j\dfrac{\omega^2 L_1 C_1 - 1}{\omega C_1}$

$\underline{Z}_2 = R_2 + j\dfrac{\omega^2 L_2 C_2 - 1}{\omega C_2}$

b) $|\underline{I}_2| = 0$ (Schalter S ist geöffnet)

c)

$$|I_1| = \frac{|\underline{U}_E|}{|\underline{Z}_1|} =$$

$$= \frac{|\underline{U}_E|}{\sqrt{R_1^2 + \left(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right)^2}}$$

$$|I_1|^2 = \frac{|U_E|^2}{R_1^2 + \left(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right)^2}$$

d) $C_{1,opt} = \arg\max |I_1|^2 =$

$$= \arg\min \underbrace{R_1^2}_{\lightning} + \underbrace{\left(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right)^2}_{\overset{!}{=}0}$$

$$\omega^2 L_1 C_1 - 1 = 0$$

$$\underline{\underline{C_1 = \frac{1}{\omega^2 L_1}}}$$

e) $\underline{Z}_1$ rein reell : $\mathrm{Im}(\underline{Z}_1) = 0$

ges: $\underline{Z}_2$, sodass $\underline{Z}_{ges} = \frac{\underline{Z}_1 \cdot \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}$ ($\underline{Z}_1$ und $\underline{Z}_2$ parallel verschalten)

$$\Rightarrow \mathrm{Im}(\underline{Z}_{ges}) = 0$$

Aus obigen Bedingungen folgt, dass bei $\mathrm{Im}(\underline{Z}_{gs}) = 0$ u. $\mathrm{Im}(\underline{Z}_1) = 0$ auch $\mathrm{Im}(\underline{Z}_2) = 0$ sein muss.

$$\mathrm{Im}(\underline{Z}_2) = 0 \Leftrightarrow \omega L_2 - \frac{1}{\omega C_2} = 0$$

$$\omega L_2 = \frac{1}{\omega C_2}$$

$$\Rightarrow C_2 = \frac{1}{\omega^2 L_2} = \frac{1}{\omega^2 2 L_1} = \frac{1}{2} \frac{1}{\omega^2 L_1} = \underline{\underline{\frac{C_1}{2}}}$$