EMF-Tutorübung, Blatt 12 – 4.2.2011

Wiederholung

$$X_{eff} = \sqrt{\frac{1}{T}\int_0^T x(t)^2\,dt} \stackrel{!}{=} \frac{\hat{X}}{\sqrt{2}}$$

$$x(t) = \hat{x}\sin(\omega t)$$

ganz allgemein

Poynting: $\left(\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^*\right)$

$$\vec{S} = \vec{E}\times\vec{H}$$

vgl. analog

Komplexe Leistung: $\boxed{\underline{S} = \frac{1}{2}\hat{\underline{u}}\,\hat{\underline{I}}^*}$

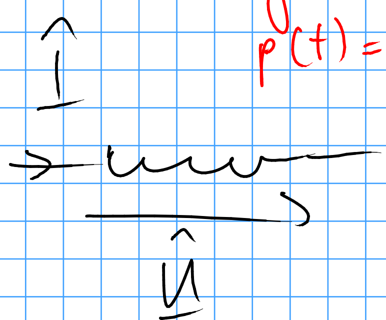
$$P_W = \mathrm{Re}(\underline{S})$$

$$P_B = \mathrm{Im}(\underline{S})$$

$\updownarrow$

Momentanleistung

$p(t) = u(t)\,i(t)$



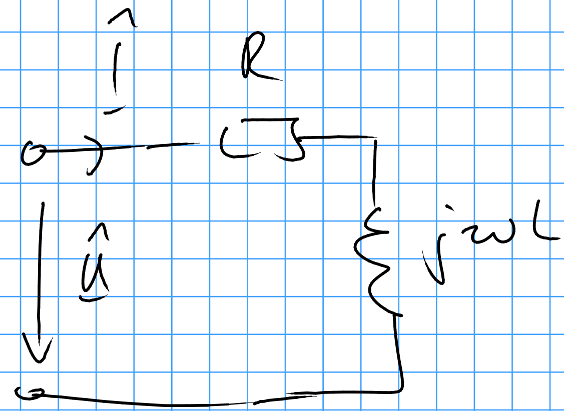
$\hat{\underline{u}} = j\omega L\hat{\underline{I}}$

$$\underline{S} = \frac{1}{2}\hat{\underline{u}}\,\hat{I}^* =$$

$$= \frac{1}{2}j\omega L\,\hat{\underline{I}}\,\hat{\underline{I}}^* = \frac{1}{2}j\omega L|\underline{\hat{I}}|^2$$

$$P_B = \frac{1}{2}\omega L\,|\underline{\hat{I}}|^2$$

Aufgabe 24:



$i(t) = \hat{I}\sin(\omega t)$
$u(t) = \hat{U}\sin(\omega t + \varphi)$

geg: $\hat{U}, \hat{I}$

ges: R, L

Lös:
$$\underline{\hat{U}} = \hat{U} e^{j\varphi_u}$$
$$\underline{\hat{I}} = \hat{I} e^{j0} = \hat{I}$$

$$\underline{Z} = R + j\omega L$$

$$\underline{Z} = \frac{\underline{\hat{U}}}{\underline{\hat{I}}} \Leftrightarrow \text{(I)}\ |\underline{Z}| = \frac{|\underline{\hat{U}}|}{|\underline{\hat{I}}|} = \frac{\hat{U}}{\hat{I}} = \sqrt{R^2 + (\omega L)^2}$$

$$\text{(II)}\quad \underline{Z} = |\underline{Z}| \cdot e^{j\varphi} = |\underline{Z}| \cdot e^{j\arg(\underline{Z})}$$

Phasenverschiebung zwischen $u(t)$ und $i(t)$ kommt durch den Imaginärteil von $\underline{Z}$ zustande

$$\varphi = \arctan\left(\frac{\mathrm{Im}(\underline{Z})}{\mathrm{Re}(\underline{Z})}\right)$$

$$\Rightarrow \frac{\omega L}{R} = \tan\frac{\pi}{4} = 1$$

$\omega L = R$ in (I)

$$|\underline{Z}| = \sqrt{R^2 + R^2} = R\sqrt{2}$$

$$\Rightarrow R = \omega L = \frac{|Z|}{\sqrt{2}} = 7{,}07\,\Omega$$

$$L = \frac{R}{\omega} = 0{,}0225\,H = \underline{22{,}5\,mH}$$

$$\sin(x) = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right)$$

$$\cos(x) = \frac{1}{2}\left(e^{jx} + e^{-jx}\right)$$

b) $p(t) = u(t)\,i(t) =$

$$= \hat{u}\sin(\omega t + \varphi)\,\hat{i}\sin(\omega t) =$$

$$= \hat{u}\hat{i}\cdot\frac{1}{2j}\left(e^{j(\omega t+\varphi)} - e^{-j(\omega t+\varphi)}\right)\cdot\frac{1}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right) =$$

$$= \frac{\hat{u}\hat{i}}{-4}\left(\underline{e^{j(2\omega t+\varphi)}} - \underline{e^{j\varphi}} - \underline{e^{-j\varphi}} + \underline{e^{-j(2\omega t+\varphi)}}\right) =$$

$$= \frac{-\hat{u}\hat{i}}{4}\left(2\cos(2\omega t+\varphi) - 2\cos(\varphi)\right) =$$

$$= \underbrace{\frac{\hat{u}\hat{i}}{2}\left(\cos(\varphi)\right.}_{\text{zeitunabhängig}} - \underbrace{\left.\cos(2\omega t+\varphi)\right)}_{\text{zeitabhängig}}$$

$\hookrightarrow P_W$

c) zeitlicher Verlauf

$$p(t) = \frac{\hat{u}\hat{i}}{2}\left(\cos(\varphi) - \cos(\underbrace{2\omega t}_{\frac{\pi}{4}} + \underset{\frac{\pi}{4}}{\varphi})\right)$$

$\longrightarrow$ Plot folgt

d) $p(t) > 0 \qquad 0 \le \omega t \le \frac{3\pi}{4}$

$p(t) < 0 \qquad -\frac{\pi}{4} \le \omega t < 0$

e) $$W^{+} = \int_{0}^{\frac{3\pi}{4\omega}} p(t)\,dt =$$

$$= \int_{0}^{\frac{3\pi}{4\omega}} \frac{\hat{u}\hat{\imath}}{2\sqrt{2}} - \frac{\hat{u}\hat{\imath}}{2}\cos(2\omega t + \varphi)\,dt =$$

$$= \left[\frac{\hat{u}\hat{\imath}}{2\sqrt{2}}t - \frac{\hat{u}\hat{\imath}}{2\cdot 2\omega}\sin(2\omega t + \varphi)\right]_{0}^{\frac{3\pi}{4\omega}} =$$

$$= \frac{\hat{u}\hat{\imath}}{2\omega}\left(\frac{3\pi}{2\sqrt{2}} + \sqrt{2}\right)$$

$$W^{-} = \int_{-\frac{\pi}{4\omega}}^{0} p(t)\,dt = \frac{\hat{u}\hat{\imath}}{2\omega}\left(\frac{\pi}{2\sqrt{2}} + \frac{\sqrt{2}}{2}\right)$$

$$W_{netto} = \frac{\hat{u}\hat{\imath}}{2\omega}\left(\frac{\pi}{\sqrt{2}} + \frac{\sqrt{2}}{2}\right)$$
$(= W^{+} - W^{-})$

f) $P_W = \mathrm{Re}\left(\frac{1}{2}\cdot\underline{\hat{U}}\,\underline{\hat{I}}^*\right) =$

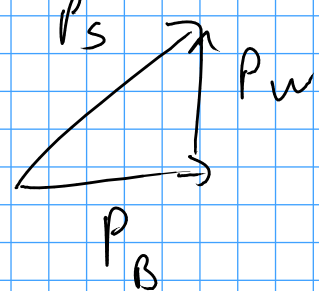
$$= \mathrm{Re}\left(\frac{1}{2}\,\hat{U}\,e^{j\frac{\pi}{4}}\,\hat{I}\right) = \frac{1}{2}\hat{U}\hat{I}\cos\left(\frac{\pi}{4}\right) =$$

$$= \frac{1}{4}\sqrt{2}\,\hat{U}\hat{I}$$

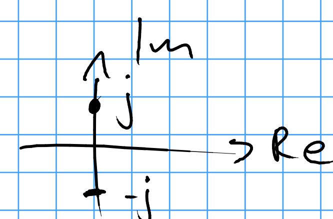
g) $P_B = \mathrm{Im}\left(\frac{1}{2}\,\underline{\hat{U}}\,\underline{\hat{I}}^*\right) = \frac{1}{2}\hat{U}\hat{I}\sin\left(\frac{\pi}{4}\right) =$

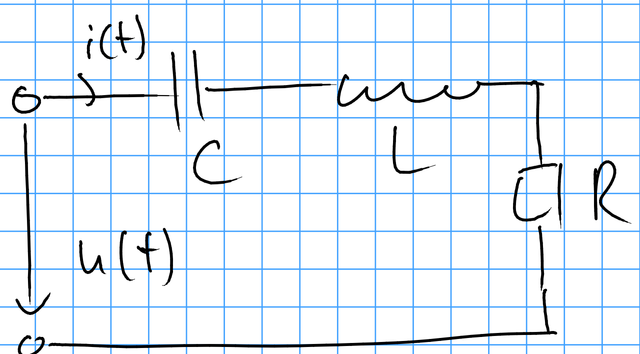
$$= \frac{1}{4}\sqrt{2}\,\hat{U}\hat{I}$$

$$P_S = \sqrt{P_B^2 + P_W^2}$$



25. Aufgabe





$\underline{\hat{I}} = \hat{I}$

$\underline{\hat{I}}_C = j\omega C\cdot\underline{\hat{U}}_C$

$\underline{\hat{U}}_C = -j\,\frac{\hat{I}_C}{\omega C} = e^{-j\frac{\pi}{2}}\,\frac{\hat{I}_C}{\omega C}$

analog: $\underline{\hat{U}}_L = j\omega L\,\hat{I}_L = e^{j\frac{\pi}{2}}\,\omega L\,\hat{I}_L$

$\underline{U}_R = R\cdot\underline{\hat{I}}$ $\Rightarrow$ keine Phasenverschiebg

b) $\underline{Z} = R + j\omega L - j\frac{1}{\omega C} =$

$= R + j\left(\omega L - \frac{1}{\omega C}\right) = R + j\,\frac{\omega^2 LC - 1}{\omega C}$

$P_W = \mathrm{Re}\left(\frac{1}{2}\,\underline{\hat{U}}\,\underline{\hat{I}}^*\right) = \mathrm{Re}\left(\frac{1}{2}\,\underline{Z}\,\underline{\hat{I}}\,\underline{\hat{I}}^*\right) = \frac{1}{2}|\underline{\hat{I}}|^2 \cdot \mathrm{Re}(\underline{Z})$

$= \frac{1}{2}|\underline{\hat{I}}|^2 \cdot R$

$P_B = \mathrm{Im}\left(\frac{1}{2}\,\underline{\hat{U}}\,\underline{\hat{I}}^*\right) = \frac{1}{2}|\underline{\hat{I}}|^2 \cdot \frac{\omega^2 LC - 1}{\omega C}$

$P_S = \sqrt{P_W^2 + P_B^2}$

c) $w_C(t) = \frac{1}{2} C u_C(t)^2$

$w_L(t) = \frac{1}{2} L i_L(t)^2$

$w_L(t) = \frac{1}{2} L \hat{I}^2 \sin^2(\omega t)$

$u_C(t) = \mathrm{Im}\left(\underline{\hat{U}}_C \cdot e^{j\omega t}\right) =$

$= \frac{\hat{I}}{\omega C} \cdot \mathrm{Im}\left(e^{-j\frac{\pi}{2}} \cdot e^{j\omega t}\right)$

$= \frac{\hat{I}}{\omega C} \sin\left(\omega t - \frac{\pi}{2}\right)$

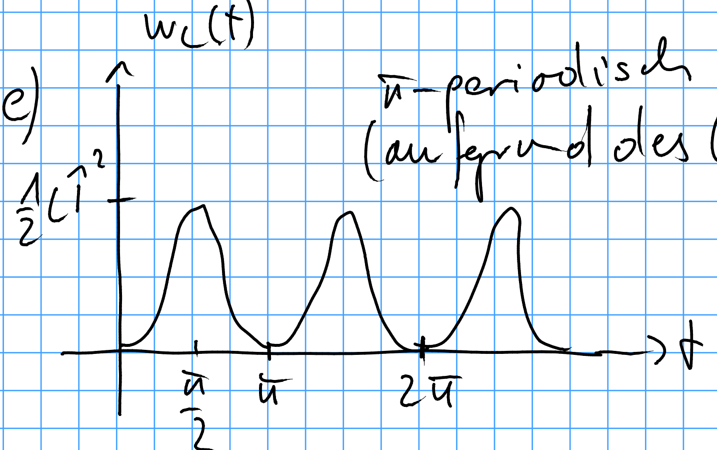
$$w_C(t) = \frac{1}{2} C \cdot \frac{\hat{I}^2}{\omega^2 C^2} \sin^2\left(\omega t - \frac{\pi}{2}\right)$$

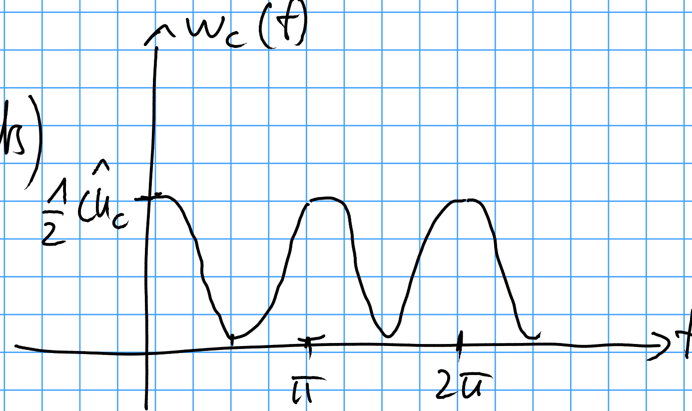
$$= \frac{1}{2} \frac{\hat{I}^2}{\omega^2 C} \cos^2(\omega t)$$

d) $W_L = \frac{1}{2} L \hat{I}^2$ ($\sin^2(\cdot)$ ist maximal 1)

$W_C = \frac{1}{2} \frac{\hat{I}^2}{\omega^2 C}$

e)





f) $P_{BL} = \mathrm{Im}\left(\frac{1}{2} \hat{\underline{U}}_L \hat{\underline{I}}_L^*\right) = \mathrm{Im}\left(\frac{1}{2} \hat{I}_L Y_L \hat{I}_L^*\right) = \frac{1}{2} \hat{I} \omega L$

$P_{BC} = \mathrm{Im}\left(\frac{1}{2} \hat{\underline{U}}_C \hat{\underline{I}}_C^*\right) = \mathrm{Im}\left(\frac{1}{2} \hat{\underline{I}}_C \frac{1}{Y_L} \cdot \hat{\underline{I}}_C^*\right) = -\frac{1}{2} \frac{\hat{I}}{\omega C}$

g) $$W_R = \int_0^{2\pi/\omega} R \hat{I}^2 \sin^2(\omega t)\, dt =$$

$$= \frac{R\hat{I}^2}{2} \int_0^{2\pi/\omega} 1 - \cos(2\omega t)\, dt$$

$$= \frac{R\hat{I}^2}{2} \left[t - \frac{1}{2\omega} \sin(2\omega t) \right]_0^{2\pi/\omega} =$$

$$= R\hat{I}^2 \frac{\pi}{2\omega}$$

h) Wert für L, sodass

$$w_L(t) + w_C(t) = \text{const.}$$

$$\frac{1}{2} L i_L^2(t) + \frac{1}{2} C u_C^2(t) = \text{const.}$$

anschauliche Bedeutung: die vorhandene Energie soll hier also zwischen der Speicherung in L oder C wechseln

mit
$$\frac{1}{2} L i_L^2(t) = \frac{1}{2} L \hat{I}^2 \sin^2(\omega t)$$
$$\frac{1}{2} C u_C^2(t) = \frac{1}{2} C \left(\frac{\hat{I}}{\omega C} \cos^2(\omega t)\right)$$

folgt:
$$\frac{1}{2} L \hat{I}^2 \sin^2(\omega t) + \frac{1}{2} \frac{C \hat{I}^2}{\omega^2 C^2} \cos^2(\omega t) = \text{const.}$$
$$L \sin^2(\omega t) + \frac{1}{\omega^2 C} \cos^2(\omega t) = \text{const}$$

$\Rightarrow$ nur dann erfüllt, falls $L = \frac{1}{\omega^2 C}$ (*), da dann die Koeffizienten vor $\sin^2(\cdot)$ und $\cos^2(\cdot)$ konstant sind, sodass der trigonometrische Pythagoras Anwendung finden kann

(*) anders formuliert ergibt ($\omega = 2\pi f$):

$$f = \frac{1}{2\pi\sqrt{LC}} \Rightarrow \text{Thompson-Formel für Resonanz}$$