EMF-Tutorübung, Blatt 9 - 14.1.2011

Wiederholung: Orthogonalentwicklung

a) Gaußsches Gesetz: $\operatorname{div} \vec{D} = \rho$ (I)

$$\operatorname{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t} = 0$$

$$\vec{E} = -\nabla \Phi \quad \text{(II)}$$

(I), (II) $\operatorname{div}(\varepsilon \nabla \Phi) = -\rho$

$$\operatorname{div}(\nabla \Phi) = -\rho/\varepsilon$$

$$\underbrace{\Delta}_{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}} \Phi = -\rho/\varepsilon$$

Vereinfacht sich hier zu:

$$\frac{\partial^2}{\partial x^2} \Phi = -\rho/\varepsilon$$

b) $\frac{\partial^2}{\partial x^2} \Phi = 0$

$\phi(x) = Ux + W$ $\qquad \frac{\partial \Phi}{\partial x} = U$

2 Randbedingung:

$x = 0$: Neumann $\Leftrightarrow \frac{\partial \Phi}{\partial x} = 0$

$x = L$: Dirichlet $\Leftrightarrow \phi(x=L) = V$

$\Rightarrow U = 0 \quad W = V \quad \phi_V(x) = V$

c) $Av = \lambda v$

$$\frac{\partial^2}{\partial x^2} b = -\lambda b \qquad b = A\cos(kx) + B\sin(kx)$$

$$\frac{\partial}{\partial x} b = -Ak\sin(kx) + Bk\cos(kx)$$

$$\frac{\partial^2}{\partial x^2} b = -Ak^2\cos(kx) - Bk^2\sin(kx)$$

$$= -k^2 b$$

durch Koeffizientenvergleich ergibt sich $\lambda = k^2$

$$\frac{\partial}{\partial x} b(x) = 0 \iff Bk = 0$$

$$\Rightarrow B = 0$$

$$b(x)\Big|_{x=L} = 0 \iff A\cos(k\cdot L) = 0$$

$$kL = \frac{\pi}{2}(2n+1) \quad n \in \mathbb{Z}$$

$$k = \frac{\pi}{2L}(2n+1)$$

Wissen, dass $B=0$ fließt hier bereits mit ein!

$$\int_0^L A^2 \cos^2(k_n x)\,dx \overset{!}{=} 1$$

$$A^2 \int_0^L \cos^2(k_n x)\,dx \overset{!}{=} 1$$

$$A^2 \left[\frac{2kL + \sin(2kx)}{4k}\right]_0^L = 1$$

$$A = \sqrt{\frac{2}{L}} \qquad \Rightarrow b_n(x) = \sqrt{\frac{2}{L}}\cos(k_n x)$$

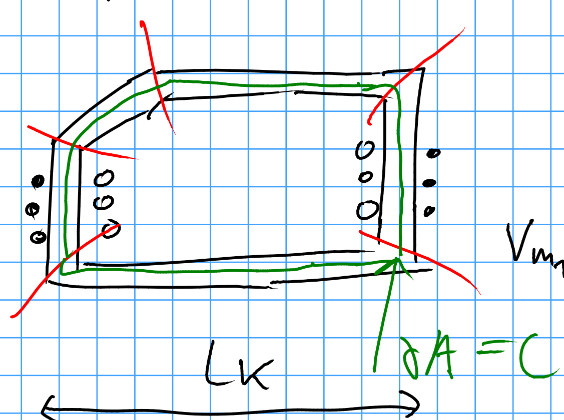
d) $$\boxed{\Phi_{hom} = \int_0^L \underbrace{\sum_{k=0}^{\infty} b_k(x)\,\frac{1}{k_n^2}\,b_k^*(x')}_{G(\vec{r},\vec{r}\,')} \left(\frac{\rho(x')}{\varepsilon}\right) dx' =}$$

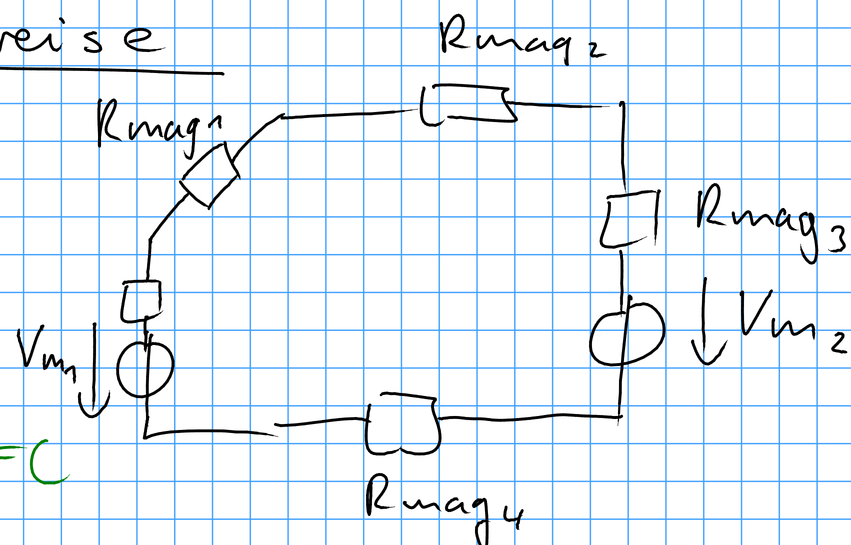
$$= \sum_{k=0}^{\infty} b_k(x)\,\frac{1}{k_n^2} \int_0^L b_k^*(x')\,\frac{\rho(x')}{\varepsilon}\,dx'$$

e) $$\Phi_{hom} = \sum_{k=0}^{\infty} b_k(x)\,\frac{1}{k_n^2\,\varepsilon}\,C \int_0^L \sqrt{\frac{2}{L}}\cos(k_n x')\,dx$$

$$\Phi = \int_V G(\vec{r},\vec{r}\,')\,\rho(\vec{r}\,')\,d^3r'$$

<u>Magnetische Kreise</u>





$$\text{rot}\,\vec{H} = j + \underbrace{\frac{\partial \vec{D}}{\partial t}}_{=0}$$

<u>Maschenregel:</u>

$$\bigcup_i \partial A_i = \partial A$$

$$\oint_{\partial A} \vec{H}\,d\vec{r} = \iint_A j\,d\vec{a}$$

$$\oint_{\partial A} \vec{H}\,d\vec{r} = \sum_k \int_{\partial A_k} \vec{H}\,d\vec{r} = \sum_k H_k \cdot L_k = \sum_k \frac{B_k}{\mu_k}\cdot L_k$$

$$= \sum_k \frac{B_k A_k}{\mu_k \cdot A_k} \cdot L_k = \sum_k \underbrace{\frac{L_k}{\mu_k A_k}}_{R_{mag}} \Phi_k$$

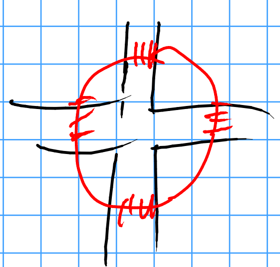
$$\left(\Phi = \iint_A \vec{B}\, d\vec{a} \qquad U_{ind} = -\frac{d\Phi}{dt} \right)$$

$$= \sum R_{mag} \Phi_k$$

$$\iint j\, d\vec{a} = \sum_i V_i = \sum_i l_i w_i$$

<u>Knotenregel:</u>

$$\bigcup_k A_k = \partial V$$



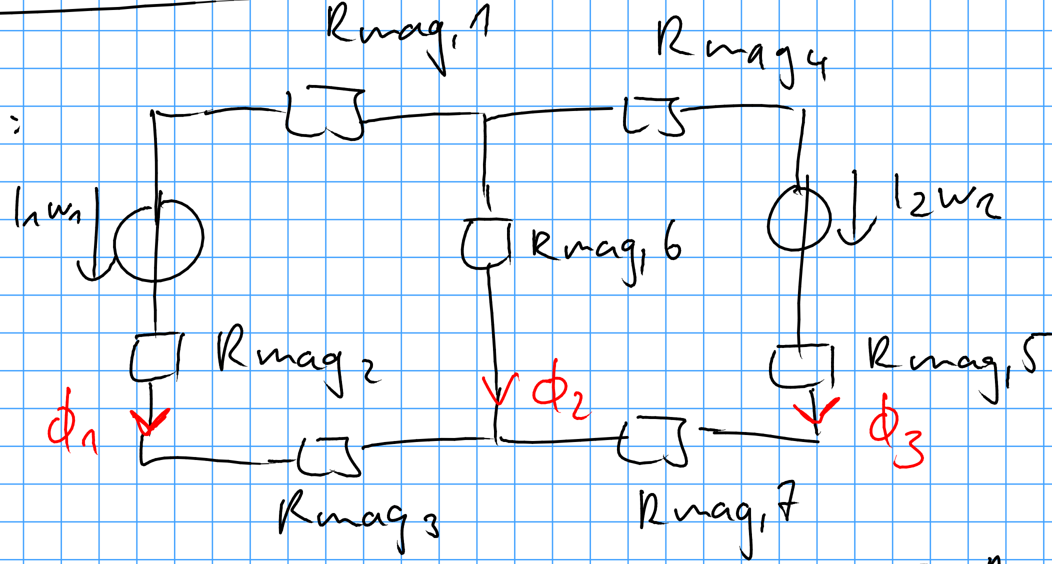
$$0 = \operatorname{div} \vec{B} = \iiint_V \operatorname{div} \vec{B}\, dV =$$

$$= \iint_{\partial V} \vec{B}\, d\vec{a} = \sum_k \iint_{A_k} \vec{B}\, d\vec{a} =$$

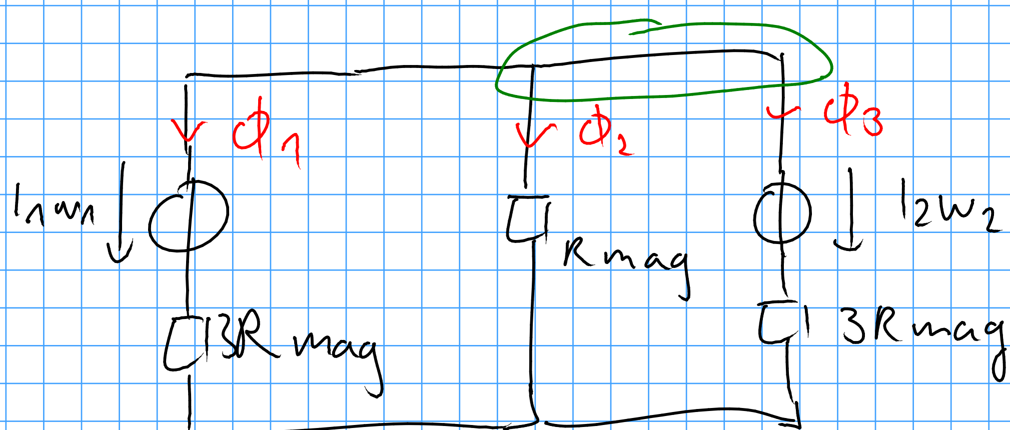
$$= \sum_k \Phi_k$$

Aufgabe 19:

ESB:



$$R_{mag,i} = \frac{l_i}{\mu_i A_i} = 3{,}97 \cdot 10^5 \, \frac{A}{Vs}$$



a) 1. Fall: $\Phi_1 = 0$

$$\Phi_2 = \frac{I_1 w_1}{R_{mag}} = -\Phi_3$$

$$I_1 w_1 = I_2 w_2 + 3R_{mag} \cdot \Phi_3$$

$$= I_2 w_2 - 3R_{mag} \cdot \frac{I_1 w_1}{R_{mag}}$$

$$= I_2 w_2 - 3 I_1 w_1$$

$$\Rightarrow I_2 = 4 I_1 = 4A$$

3. $\Phi_3 = 0$

$$\Phi_2 = \frac{I_2 w_2}{R_{mag}} = -\Phi_1$$

$$I_2 w_2 = I_1 w_1 - 3 R_{mag} \frac{I_2 w_2}{R_{mag}}$$

$$4 I_2 = I_1$$

$$I_2 = \frac{1}{4} A$$

2. $\Phi_2 = 0$

$\hookrightarrow \quad \Phi_1 = -\Phi_3$

$\hookrightarrow \quad I_1 w_1 = -I_2 w_2 \quad \Rightarrow \quad I_2 = -1A$

b) $$\Phi_2 \Big|_{\Phi_1 = 0} = \frac{I_1 w_1}{R_{mag}} = 2{,}52 \cdot 10^{-4} \, Vs$$

⋮