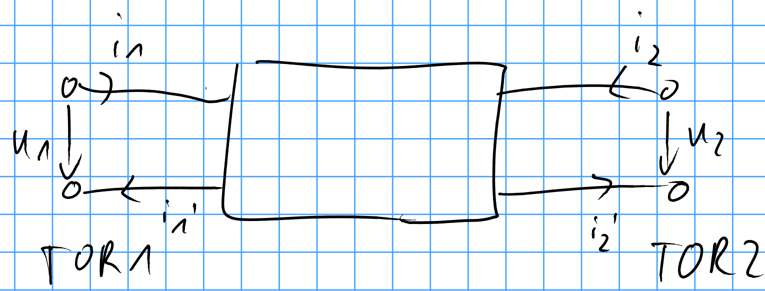
ST 1 - Tutorübung Blatt 4



$i_1 = i_1'$

$i_2 = i_2'$

Beschreibungsformen:

→ explizit:

- Widerstandsbeschreibung: $\underline{u} = \underset{\sim}{R}\,\underline{i}$
- Leitwertsbeschreibung: $\underline{i} = \underset{\sim}{G}\,\underline{u}$
- Hybridbeschreibung: $\begin{pmatrix} u_1 \\ i_2 \end{pmatrix} = \underset{\sim}{H} \begin{pmatrix} i_1 \\ u_2 \end{pmatrix}$
- inverse Hybridmatrix $\begin{pmatrix} i_1 \\ u_2 \end{pmatrix} = \underset{\sim}{H}' \begin{pmatrix} u_1 \\ i_2 \end{pmatrix}$
- Kettenmatrix $\begin{pmatrix} u_1 \\ i_1 \end{pmatrix} = \underset{\sim}{A} \cdot \begin{pmatrix} u_2 \\ -i_2 \end{pmatrix}$
- inverse Kettenmatrix $\begin{pmatrix} u_2 \\ i_2 \end{pmatrix} = \underset{\sim}{A}' \cdot \begin{pmatrix} u_1 \\ -i_1 \end{pmatrix}$

⇒ Darstellung in Matrix/Vektorschreibweise nur für lineare Zweitore möglich!

⇒ Aufstellen einer solchen Matrix

↳ „by inspection" z.B. ges: $\underset{\sim}{R}$ $\quad \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \underset{\sim}{R} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}$

$u_1 = f_1(i_1, i_2)$

$u_2 = f_2(i_1, i_2)$

↳ LL/KS-Methode

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \underset{\sim}{R} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}$$

$$= \begin{pmatrix} r_{11} i_1 + r_{12} i_2 \\ r_{21} i_1 + r_{22} i_2 \end{pmatrix}$$

$$\Rightarrow r_{11} = \frac{u_1}{i_1} \bigg|_{i_2 = 0A} \qquad r_{21} = \frac{u_2}{i_1} \bigg|_{i_2 = 0A}$$

$$\Rightarrow r_{12} = \frac{u_1}{i_2} \bigg|_{i_1 = 0A} \qquad r_{22} = \frac{u_2}{i_2} \bigg|_{i_1 = 0A}$$

→ implizite Beschreibung (Nullstellenmenge, Kernbeschreibung)

$$\begin{pmatrix} \underset{\sim}{M} & \underset{\sim}{N} \end{pmatrix} \cdot \begin{pmatrix} \underline{u} \\ \underline{i} \end{pmatrix} = 0$$

$$\underset{\sim}{M} \underline{u} + \underset{\sim}{N} \underline{i} = 0$$

$$\underline{i} = \underset{\sim}{G} \underline{u} \quad | - \underset{\sim}{G} \underline{u}$$

$$\underline{i} - \underset{\sim}{G} u = 0$$

$$\underset{\sim}{N} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \underset{\sim}{M} = -\underset{\sim}{G}$$

→ parametrisierte Beschreibung:

$$\begin{pmatrix} \underline{u} \\ \underline{i} \end{pmatrix} = \begin{pmatrix} \underset{\sim}{U} \\ \underset{\sim}{I} \end{pmatrix} \cdot \underline{c} = \begin{pmatrix} u_1^{(1)} & u_1^{(2)} \\ u_2^{(1)} & u_2^{(2)} \\ i_1^{(1)} & i_1^{(2)} \\ i_2^{(1)} & i_2^{(2)} \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

1. Messung   2. Messung

$u_1^{(1)} = 1V$ $u_1^{(2)} = 0V$

$u_2^{(1)} = 0V$ $u_2^{(2)} = 1V$

Eigenschaften

→ verlustlos: $\forall t \quad p_1(t) + p_2(t) = 0 \Leftrightarrow \underline{u}^T \cdot \underline{i} = 0$

$u_1 \cdot i_1 + u_2 i_2$

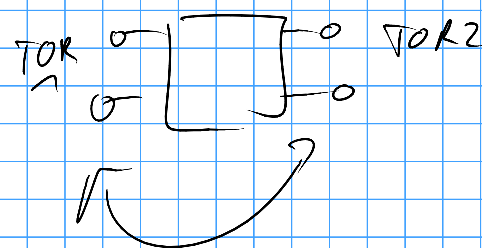
→ aktiv/passiv: aktiv: $\underline{u}^T \cdot \underline{i} < 0$ aktiv

$\underline{u}^T \cdot \underline{i} \geq 0$ passiv

→ Umkehrbarkeit: $\underset{\sim}{R} = \underset{\sim}{P}\,\underset{\sim}{R}\,\underset{\sim}{P} \qquad \underset{\sim}{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\underset{\sim}{G} = \underset{\sim}{P}\,\underset{\sim}{G}\,\underset{\sim}{P}$

$\underset{\sim}{H} = \underset{\sim}{P}\,\underset{\sim}{H}'\,\underset{\sim}{P}$



→ Reziprozität: $\underset{\sim}{G} = \underset{\sim}{G}^T$

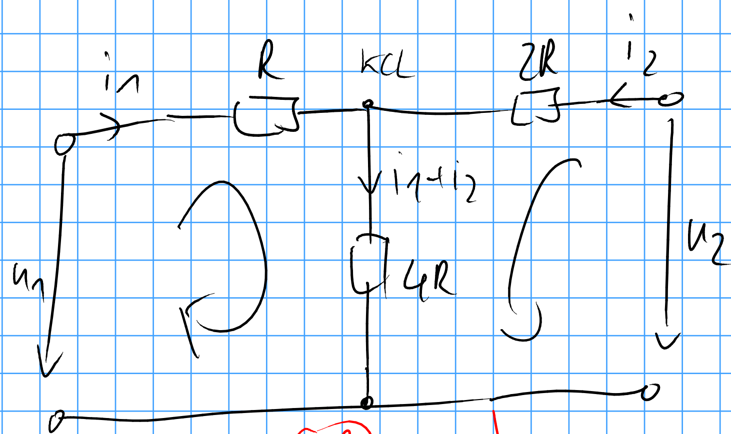
$\underset{\sim}{R} = \underset{\sim}{R}^T$

$\det \underset{\sim}{A} = \det \underset{\sim}{A}' = 1$

$A \cdot A^{-1} = I_n$

$A (A')^{-1} \neq I_n$

Blatt 4



1. $\underline{u} = \underset{\sim}{R}\,\underline{i}$

$u_1 = R i_1 + 4R(i_1 + i_2)$

$= R i_1 + 4R i_1 + 4R i_2$

$= 5R i_1 + 4R i_2$

$u_2 = 2R i_2 + 4R(i_1 + i_2)$

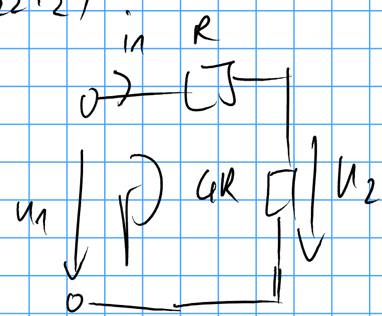
$= 4R i_1 + 6R i_2$

$\Rightarrow \underset{\sim}{R} = \begin{pmatrix} 5R & 4R \\ 4R & 6R \end{pmatrix}$

LL/KS-Methode:

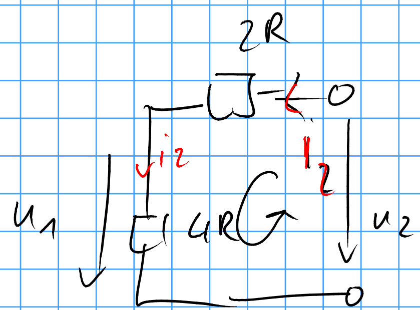
$$\underline{u} = \underset{\sim}{R}\,\underline{i} = \begin{pmatrix} r_{11} i_1 + r_{12} i_2 \\ r_{21} i_1 + r_{22} i_2 \end{pmatrix}$$

$$\Rightarrow r_{11} = \frac{u_1}{i_1}\bigg|_{i_2 = 0A}$$



$$u_1 = R i_1 + 4R i_1$$

$$r_{11} = \frac{5R i_1}{i_1} = 5R$$



$$r_{12} = \frac{u_1}{i_2}\bigg|_{i_1 = 0A} = 4R$$

$$\parallel$$

$$r_{21} = \frac{u_2}{i_1}\bigg|_{i_2 = 0A} = 4R$$

$$r_{22} = \frac{u_2}{i_2}\bigg|_{i_1 = 0A} = \frac{2R i_2 + 4R i_2}{i_2} = \underline{\underline{6R}}$$

2. $\Rightarrow$ passiv, $\underline{u}^T \cdot \underline{i} \geq 0$ ✓

$\Rightarrow$ spannungsgesteuert

$\Rightarrow$ stromgesteuert

$\Rightarrow$ Reziprozität

$$\underset{\sim}{R} = R^T$$

$$\underset{\sim}{R} = \begin{pmatrix} 5R & 4R \\ 4R & 6R \end{pmatrix} \quad \underset{\sim}{R}^T = \begin{pmatrix} 5R & 4R \\ 4R & 6R \end{pmatrix}$$

✓ $$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 5R & 4R \\ 4R & 6R \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}$$

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \frac{1}{14R} \begin{pmatrix} 6 & -4 \\ -4 & 5 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

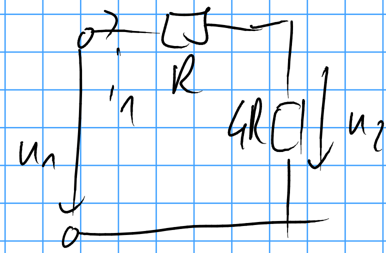
$$\underline{u}^T \cdot \underline{i} = u_1 i_1 + u_2 i_2$$
$$= (5R i_1 + 4R i_2) i_1 + (4R i_1 + 6R i_2) i_2$$
$$= 5R i_1^2 + 4R i_1 i_2 + 4R i_1 i_2 + 6R i_2^2$$
$$= 5R i_1^2 + 8R i_1 i_2 + 6R i_2^2$$

$\boxed{\text{Trick}}$ $$= R\left[\left(\sqrt{5}\, i_1 + \frac{4}{\sqrt{5}} i_2\right)^2 - \frac{16}{5} i_2^2 + 6 i_2^2\right] > 0$$

3. Kettenmatrix

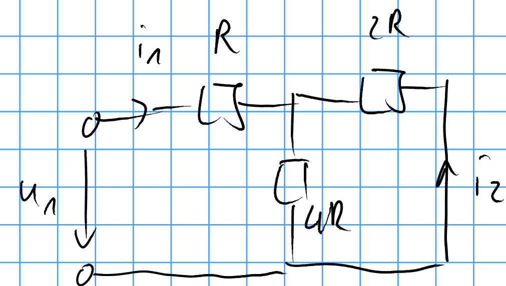
$$\begin{pmatrix}u_1\\ i_1\end{pmatrix} = \underset{\sim}{A}\begin{pmatrix}u_2\\ -i_2\end{pmatrix} = \begin{pmatrix} a_{11}u_2 - a_{12}i_2 \\ a_{21}i_1 - a_{22}i_2\end{pmatrix} \qquad u_1 = a_{11}u_2$$

$$a_{11} = \left.\frac{u_1}{u_2}\right|_{i_2 = 0A}$$



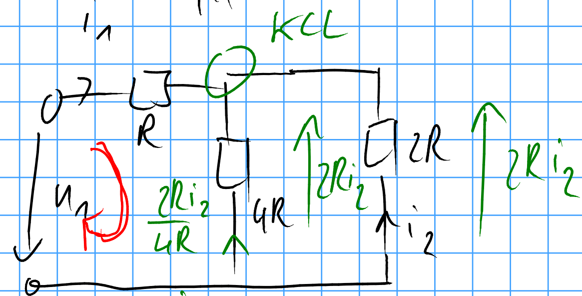
Spgteiler: $u_2 = u_1 \cdot \frac{4R}{5R} = u_1 \cdot \frac{4}{5}$

$$a_{11} = \frac{u_1}{u_1 \cdot \frac{4}{5}} = \frac{5}{4}$$



$$a_{12} = -\left.\frac{u_1}{i_2}\right|_{u_2 = 0V}$$

$$= -\frac{-3,5Ri_2}{i_2} = 3,5R$$



$= \frac{i_2}{2}$

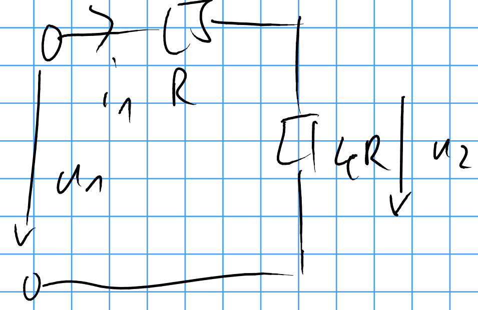
KCL: $-i_1 - \frac{i_2}{2} - i_2 = 0$

$i_1 = -\frac{3}{2}i_2$

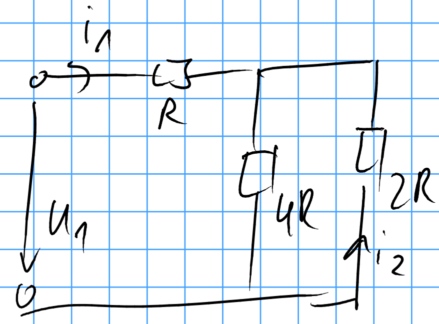
KVL: $u_1 = R\left(-\frac{3}{2}i_2\right) - 2Ri_2 =$

$= -\frac{3}{2}Ri_2 - 2Ri_2 = -3,5Ri_2$

$$a_{21} = \left.\frac{i_1}{u_2}\right|_{i_2=0} = \frac{1}{4R}$$



$$a_{22} = -\left.\frac{i_1}{i_2}\right|_{u_2=0}$$



Stromteiler: $i_2 = -i_1 \cdot \frac{\frac{1}{2R}}{\frac{1}{2R}+\frac{1}{4R}} = -i_1 \cdot \frac{\frac{1}{2R}}{\frac{3}{4R}} = -i_1 \cdot \frac{1}{2R} \cdot \frac{4R}{3}$

$= -\frac{2}{3} i_1$

$$a_{22} = \frac{-i_1}{-\frac{2}{3} i_1} = \underline{\underline{\frac{3}{2}}}$$

4. $\underset{\sim}{G} = \underset{\sim}{R}^{-1}$ Erinnerung: geg: $\underset{\sim}{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Lös: $\underset{\sim}{A}^{-1} = \frac{1}{\det A} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$$\underset{\sim}{G} = \frac{1}{\det \underset{\sim}{R}} \begin{pmatrix} r_{22} & -r_{12} \\ -r_{21} & r_{11} \end{pmatrix}$$

$$\underset{\sim}{R} = \begin{pmatrix} 5R & 4R \\ 4R & 6R \end{pmatrix} \quad \det \underset{\sim}{R} = 30R^2 - 16R^2 = 14R^2$$

$$\underset{\sim}{G} = \frac{1}{14R^2} \begin{pmatrix} 6R & -4R \\ -4R & 5R \end{pmatrix} = \underline{\underline{\begin{pmatrix} \frac{3}{7R} & -\frac{2}{7R} \\ -\frac{2}{7R} & \frac{5}{14R} \end{pmatrix}}}$$

5. Implizite Beschreibung

$$\begin{pmatrix} \underset{\sim}{M} & \underset{\sim}{N} \end{pmatrix} \cdot \begin{pmatrix} \underline{u} \\ \underline{i} \end{pmatrix} = 0 \iff \underset{\sim}{M}\,\underline{u} + \underset{\sim}{N}\,\underline{i} = 0$$

ausgehend von $\underline{u} = \underset{\sim}{R}\,\underline{i}$ ergibt sich:

$$u - \underset{\sim}{R}\, i = 0$$

Koeffizientenvergleich liefert:

$$\underset{\sim}{M} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \underset{\sim}{1}_2$$

$$\underset{\sim}{N} = -\underset{\sim}{R}$$

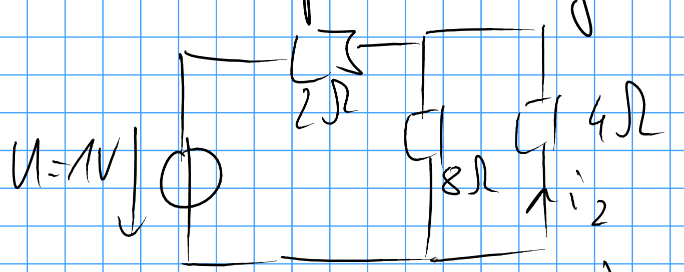
6. parametrisierte Beschreibung

$$\begin{pmatrix} \underline{u} \\ \underline{i} \end{pmatrix} = \begin{pmatrix} \underset{\sim}{U} \\ \underset{\sim}{I} \end{pmatrix} \cdot \underline{c} = \begin{pmatrix} u_1^{(1)} & u_1^{(2)} \\ u_2^{(1)} & u_2^{(2)} \\ i_1^{(1)} & i_1^{(2)} \\ i_2^{(1)} & i_2^{(2)} \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$\left.\begin{aligned} &\text{Beschaltung Messung 1: } u_1^{(1)} = 1\,V \\ &\phantom{\text{Beschaltung Messung 1: }} u_2^{(1)} = 0\,V \\ &\text{Beschaltung Messung 2: } u_1^{(2)} = 0\,V \\ &\phantom{\text{Beschaltung Messung 2: }} u_2^{(2)} = 1\,V \end{aligned}\right\}$$

Werte frei gewählt, jedoch darauf achten, dass diese linear unabhängig sein müssen

ESB für Messung 1:



mit Hilfe von $\underline{i} = \underset{\sim}{G}\,\underline{u}$ ergibt sich:

$i_1^{(1)} = \frac{3}{14}A \qquad i_2^{(1)} = -\frac{2}{14}A$

$i_1^{(2)} = -\frac{2}{14}A \qquad i_2^{(2)} = \frac{5}{28}A$

$$\Rightarrow \begin{pmatrix} \underset{\sim}{U} \\ \underset{\sim}{I} \end{pmatrix} = \begin{pmatrix} 1V & 0V \\ 0V & 1V \\ 3/14A & -2/14A \\ -2/14A & 5/28A \end{pmatrix}$$