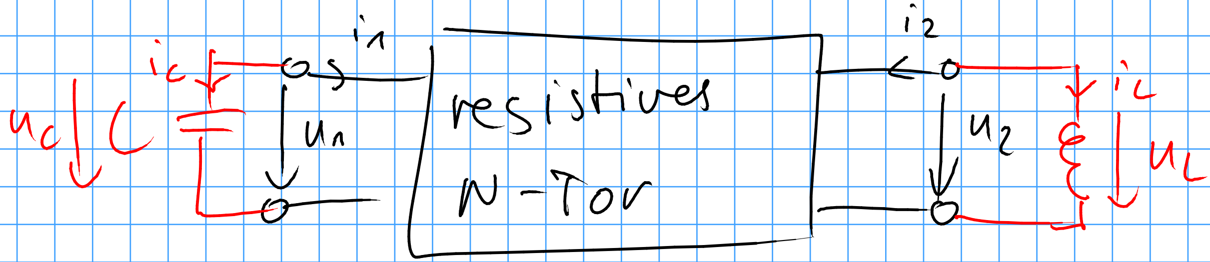
ST2 - Tutorübung - Blatt 3, 24.5.2011

Lineare Schaltungen 2. Grades

→ mind. 2 reaktive Elemente treten auf



$$i_C = C\dot{u}_C \qquad \dot{u}_1 = \frac{1}{C} i_C = -\frac{1}{C} i_1 = f(u_1, i_2)$$

$$u_L = L\dot{i}_L \qquad \dot{i}_2 = -\frac{u_2}{L} = f(u_1, i_2)$$

$$\dot{i}_L = \frac{1}{L} u_L \Leftrightarrow -\dot{i}_2 = \frac{1}{L} u_2$$

Ausgangssituation:

$$i_1 = f(u_1, i_2)$$

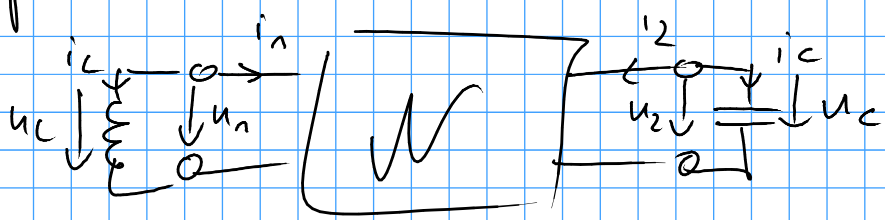
$$u_2 = f(u_1, i_2)$$

⇒ inverse Hybridmatrix

$$\begin{pmatrix} i_1 \\ u_2 \end{pmatrix} = \underset{\sim}{H}' \begin{pmatrix} u_1 \\ i_2 \end{pmatrix}$$

$$\begin{pmatrix} \dot{u}_1 \\ \dot{i}_2 \end{pmatrix} = \underbrace{\begin{pmatrix} -\frac{1}{C} & 0 \\ 0 & -\frac{1}{L} \end{pmatrix} \underset{\sim}{H}'}_{\underset{\sim}{A}} \begin{pmatrix} u_1 \\ i_2 \end{pmatrix}$$

2. Beispiel

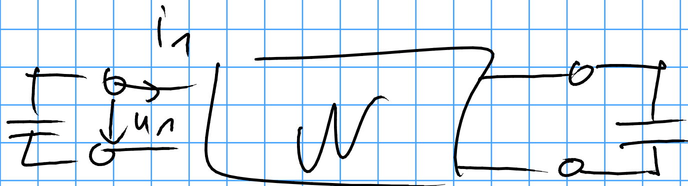


$$\dot{i}_1 = -\frac{u_1}{L} = f(i_1, u_2)$$

$$\dot{u}_2 = -\frac{i_2}{C} = f(i_1, u_2)$$

$\Rightarrow$ Hybridmatrix

3.

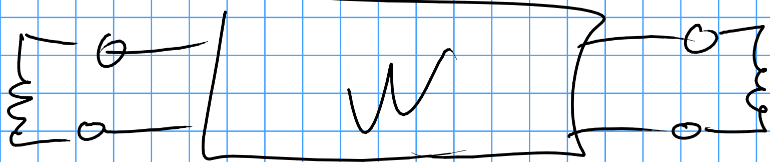


$$\dot{u}_1 = -\frac{1}{C} i_1 = f(u_1, u_2)$$

$$\dot{u}_2 = -\frac{1}{C} i_2 = f(u_1, u_2)$$

$\Rightarrow$ Leitwertsbeschreibung

4.



$\Rightarrow$ Widerstandsbeschreibung

Ergebnis dieses Vorgehens ist eine lineare DGL:

$$(*) \quad \dot{\underline{x}} = \underset{\sim}{A}\,\underline{x} \qquad \text{z.B. } x = \begin{pmatrix} u_1 \\ i_2 \end{pmatrix}$$

bezogen auf Fall 1

skalarer Fall: $\dot{x}(t) = \frac{1}{\tau} x(t)$

allg. $x(t) = C e^{t/\tau}$

Auch für (*) gilt: $\underline{x} = \underbrace{e^{t\underset{\sim}{A}}}_{\text{Matrixexponential}} \underline{x}_0$

$$\exp(x) = \sum_{k=0}^{\infty} \frac{x^{k!}}{k!}$$

$$\exp(t\underset{\sim}{A}) = \sum_{k=0}^{\infty} \frac{(tA)^k}{k!} \quad ;$$

Diagonalisierung:

$$\underset{\sim}{A} = Q \Lambda Q^{-1}$$

$$\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \dots, \lambda_n)$$

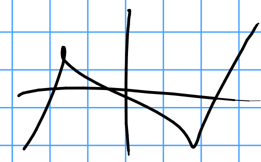
$$Q = (q_1, q_2, q_3, \dots, q_n)$$

$$(tA)^k = (t Q \Lambda Q^{-1})^k = t^k (Q \Lambda Q^{-1})^k =$$

$$= t^k Q \Lambda \underbrace{Q^{-1} Q} \Lambda Q^{-1} \dots$$

$$= t^k Q \underset{\sim}{\Lambda}^k Q^{-1}$$

Aufgabe 3



1) Negativer Immitanzkonverter

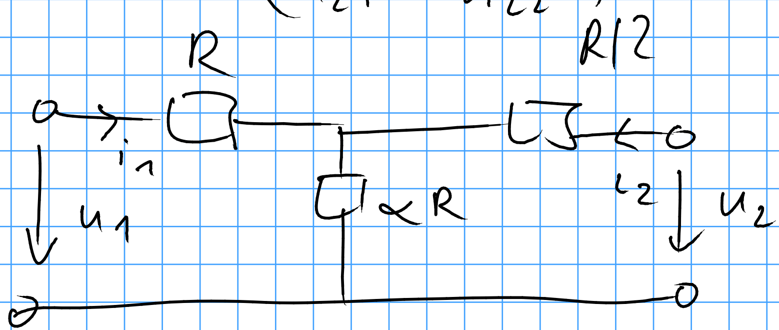
2) Zustandsgrößen: u_1, i_2

$$\dot{u}_1 = -\frac{1}{C} i_1 = f(u_1, i_2)$$

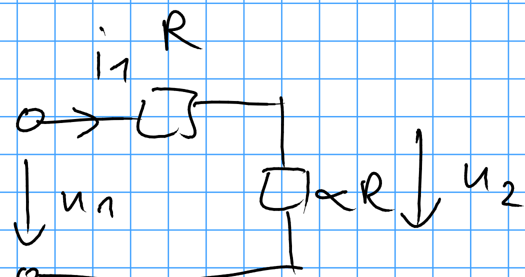
$$-\dot{i}_2 = \frac{1}{L} u_2 = f(u_1, i_2)$$

$\rightarrow$ inverse Hybridmatrix

3) $\underset{\sim}{H}' = \begin{pmatrix} h_{11}' & h_{12}' \\ h_{21}' & h_{22}' \end{pmatrix}$ $\qquad \begin{pmatrix} i_1 \\ u_2 \end{pmatrix} = \underset{\sim}{H}' \begin{pmatrix} u_1 \\ i_2 \end{pmatrix}$

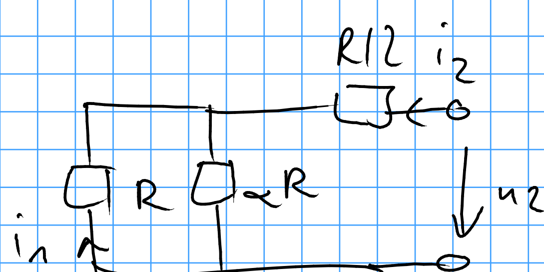


LL/KS-Methode



$$h_{11}' = \frac{i_1}{u_1}\bigg|_{i_2=0}$$

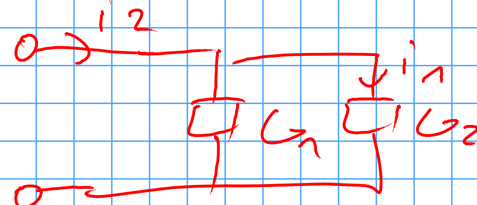
$$= \frac{1}{R + \alpha R}$$



$$h_{12}' = \frac{i_1}{i_2}\bigg|_{u_1=0}$$

Stromteiler: $i_1 = -i_2 \cdot \dfrac{\frac{1}{R}}{\frac{1}{R} + \frac{1}{\alpha R}}$

$$= -\frac{\alpha}{1+\alpha}$$



$$i_1 = i_2 \cdot \frac{G_2}{G_1+G_2}$$

$$h'_{21} = \frac{u_2}{u_1}\Big|_{i_2=0} \qquad u_2 = u_1 \cdot \frac{\alpha R}{R+\alpha R} = \frac{\alpha}{1+\alpha}$$

$$= \frac{\alpha}{1+\alpha}$$

$$h'_{22} = \frac{u_2}{i_2}\Big|_{u_1=0} = \frac{R}{2} + \alpha R \| R = \frac{R}{2} + \frac{\alpha R}{1+\alpha}$$

Systemmatrix: $$\underset{\sim}{A} = \begin{pmatrix} -1/C & 0 \\ 0 & -1/L \end{pmatrix} \underset{\sim}{H}' \overset{!}{=}$$

$$= -\begin{pmatrix} \frac{1}{C(R+\alpha R)} & -\frac{\alpha}{C(1+\alpha)} \\ \frac{\alpha}{L(1+\alpha)} & \frac{1}{C}\left(\frac{R}{2}+\frac{\alpha R}{1+\alpha}\right) \end{pmatrix}$$

$$\dot{\underline{x}} = \underset{\sim}{A}\,\underline{x} \qquad \underline{x} = \begin{pmatrix} u_1 \\ i_2 \end{pmatrix}$$

4. $$\underset{\sim}{A} = \begin{pmatrix} -\frac{1}{\alpha+1} & \frac{\alpha}{1+\alpha} \\ \frac{-2\alpha}{\alpha+1} & -1-\frac{2\alpha}{\alpha+1} \end{pmatrix} = \Big|_{\alpha=-\frac{3}{4}}$$

$$= \begin{pmatrix} -4 & -3 \\ 6 & 5 \end{pmatrix}$$

Eigenvektoren / Eigenwerte

$\underset{\sim}{A}\,\underline{v} = \lambda \underline{v}$

Vektoren, die neben-stehende Gleichung erfüllen, nennt man EV. Die dazugehörigen λ bezeichnet man als Eigenwerte.

$A \in \mathbb{R}^{n \times n}$

können max. n-verschiedene EW besitzen

$\underset{\sim}{A} v - \lambda \underline{v} = 0$

$(\underset{\sim}{A} - \lambda \underset{\sim}{E_n})\,\underline{v} = 0$ ges: $\underline{v}$, jedoch $\underline{v} \neq \underline{0}$

$(\underset{\sim}{A} - \lambda E_n)$ nicht invertierbar, sonst nur triviale Lösung existent

$\det(A - \lambda E_n) \overset{!}{=} 0$

Beispiel: $\underset{\sim}{A} = \begin{pmatrix} -4 & -3 \\ 6 & 5 \end{pmatrix}$

$\det(A - \lambda E_2) = \begin{vmatrix} -4-\lambda & -3 \\ 6 & 5-\lambda \end{vmatrix} =$

$= (-4-\lambda)(5-\lambda) + 18 \overset{!}{=} 0$

$\underline{-20} + 4\lambda - 5\lambda + \underline{\lambda^2} + \underline{18} = 0$

$$\lambda^2 - \lambda - 2 = 0$$

$$\left.\begin{aligned} \lambda_1 &= -1 \\ \lambda_2 &= 2 \end{aligned}\right\}$$ EW der Zustandsmatrix $\underset{\sim}{A}$

EV als Lösungen von $(A - \lambda E_2)v \overset{!}{=} 0$

$\lambda_1 = -1$

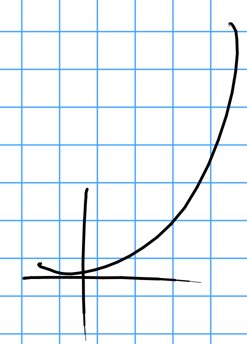
$$\begin{pmatrix} -3 & -3 \\ 6 & 6 \end{pmatrix} \underline{v_1} = 0 \quad \Rightarrow v_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$\lambda_2 = 2$

$$\begin{pmatrix} -6 & -3 \\ 6 & 3 \end{pmatrix} \underline{v_2} = 0 \quad \Rightarrow v_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

6. Schaltung ist dann stabil, falls

$\mathrm{Re}(\lambda_n) < 0$ $\qquad \sum_i c_i e^{\lambda_i t}$



$$\lambda_2 = -\frac{2\alpha + 1}{1 + \alpha} < 0 \quad | \cdot (-1)$$

$$\frac{2\alpha + 1}{1 + \alpha} > 0$$

1. Fall: $2\alpha + 1 > 0 \quad \wedge \quad 1 + \alpha > 0$

$\boxed{\alpha > -\frac{1}{2}} \qquad \alpha > -1$

2. Fall $2\alpha + 1 < 0 \quad \wedge \quad 1 + \alpha < 0$

$\alpha < -\frac{1}{2} \qquad \boxed{\alpha < -1}$

$$\alpha_{\text{stabil}} \in \left(-\infty ; -1\right) \cup \left[-\frac{1}{2} ; +\infty\right)$$

Teilaufgaben 5|7 werden zu einem späteren Zeitpunkt nochmals ausführlicher behandelt.