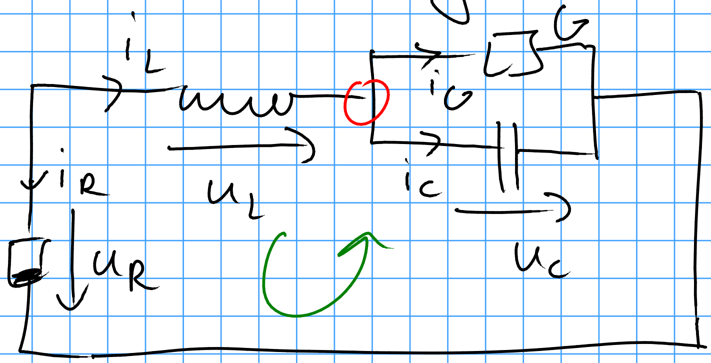
ST2-Tutorübung, Blatt 8, 28.06.2011



1. $\dot{\underline{x}} = f(\underline{x})$

$$\underline{x} = \begin{pmatrix} u_C \\ i_L \end{pmatrix}$$

$$\dot{u}_C = \frac{1}{C} i_C = \frac{1}{C} i_C(u_C, i_L)$$

$$\dot{i}_L = \frac{1}{L} u_L = \frac{1}{L} u_L(u_C, i_L)$$

Knoten: $i_L = i_G + i_C = u_C \cdot G + i_C$

$$\Rightarrow i_C = i_L - u_C \cdot G$$

$$\Rightarrow \dot{u}_C = \frac{1}{C}\left(i_L - u_C \cdot G\right)$$

Schleife: $u_L = u_R - u_C =$

$$= -d\, i_R\, \Omega + \left(\frac{i_R}{A}\right)^3 \cdot V - u_C =$$

$$= d\, i_L\, \Omega - \left(\frac{i_L}{A}\right)^3 V - u_C$$

$$\Rightarrow \dot{i}_L = \frac{1}{L}\left(d\, i_L\, \Omega - \left(\frac{i_L}{A}\right)^3 V - u_C\right)$$

2. stromgesteuert, da der AP eindeutig durch i_R festgelegt wird

3. GGP | Fixpunkte bestimmen

$$\begin{cases} \dot{x}_1 = 0 \\ \dot{x}_2 = 0 \end{cases} \qquad \begin{cases} -3x_1 + 3x_2 = 0 \\ -2x_1 + 4x_2 - 2x_2^3 = 0 \end{cases}$$

(I^*) $x_1 = x_2$ in II

$$-2x_2 + 4x_2 - 2x_2^3 = 0$$

$$x_2^2 = 1 \quad \vee \quad x_2 = 0$$

$$x_2 = \pm 1$$

$GGP_1\ (0, 0)$

$GGP_2\ (1, 1)$

$GGP_3\ (-1, -1)$

4. Linearisieren des System: Jacobi-Matrix

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_2} \end{pmatrix} = \begin{pmatrix} -3 & 3 \\ -2 & 4 - 6x_2^2 \end{pmatrix}$$

$$J\Big|_{GGP_1} = \begin{pmatrix} -3 & 3 \\ -2 & 4 \end{pmatrix}$$

$$\det(\lambda E_2 - J) \overset{!}{=} 0$$

$$\begin{vmatrix} \lambda+3 & -3 \\ 2 & \lambda-4 \end{vmatrix} \overset{!}{=} 0 \iff (\lambda+3)(\lambda-4)+6 \overset{!}{=} 0$$

$$\lambda^2 - 4\lambda + 3\lambda - 12 + 6 \overset{!}{=} 0$$

$$\lambda^2 - \lambda - 6 = 0$$

$$\lambda_1 = -2$$

$$\lambda_2 = 3$$

6. Sattelpunkt $\quad \lambda_1 < 0$

$$\lambda_2 > 0$$

7. $$J\Big|_{GGP_1, GGP_2} = \begin{pmatrix} -3 & 3 \\ -2 & -2 \end{pmatrix}$$

$$\det(\lambda \underset{\sim}{E}_2 - J) \overset{!}{=} 0 \iff \begin{vmatrix} \lambda+3 & -3 \\ 2 & \lambda+2 \end{vmatrix} \overset{!}{=} 0$$

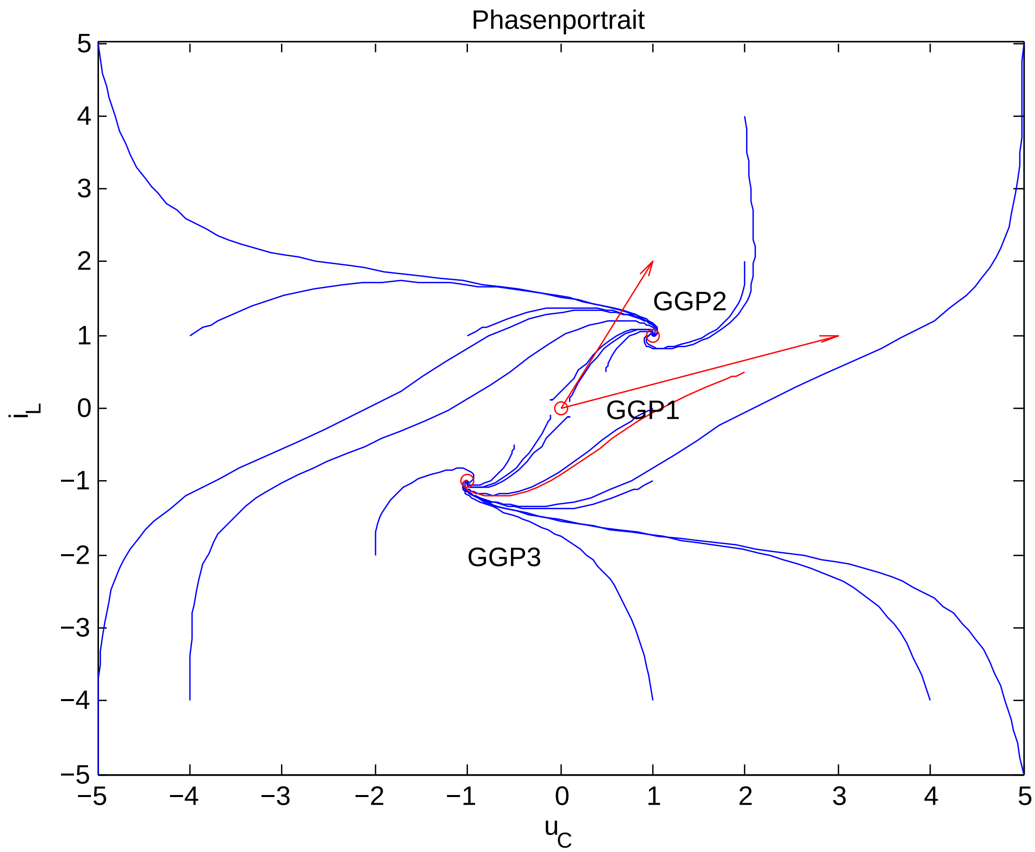
$$(\lambda+3)(\lambda+2)+6 \overset{!}{=} 0$$

$$\lambda^2 + 5\lambda + 12 = 0$$

$$\lambda_{1/2} = \frac{-5 \pm \sqrt{25-48}}{2} =$$

$$= -\frac{5}{2} \pm j\frac{\sqrt{23}}{2}$$ $\Rightarrow$ stabiler Strudel

8. Speicher: bistabile Schaltung, aka FlipFlop



Phasenportrait
GGP1
GGP2
GGP3
u_C
i_L