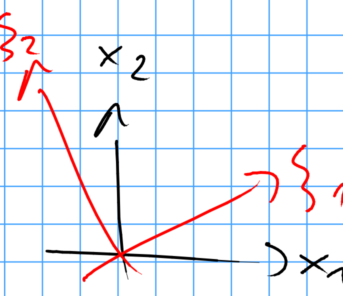
ST2 - Tutorübung, Blatt 9

Wiederholung: Zustandsebene / ξ-Ebene

$\dot{\underline{x}} = \underset{\sim}{A}\, \underline{x}$ o.B.d.A homogener Fall

$\dot{\underline{x}} = \underset{\sim}{Q}\,\underset{\sim}{\Lambda}\,\underset{\sim}{Q}^{-1}\underline{x} \quad | \to \underset{\sim}{Q}^{-1}$



$\underset{\sim}{Q}^{-1}\dot{\underline{x}} = \underbrace{\underset{\sim}{Q}^{-1}\underset{\sim}{Q}}_{\underset{\sim}{E}_n}\,\underset{\sim}{\Lambda}\,\underbrace{\underset{\sim}{Q}^{-1}\underline{x}}_{\xi}$

$\dot{\xi} = \underset{\sim}{\Lambda}\,\xi \qquad \xi = \underset{\sim}{Q}^{-1}\underline{x} \iff \underline{x} = \underset{\sim}{Q}\,\xi$

$\hookrightarrow$ entkoppelt, da $\underset{\sim}{\Lambda} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$

Sonderfall: $\lambda_1, \lambda_2 \in \mathbb{C}$

$\lambda_1 = \lambda_2^* = \lambda = \alpha + j\beta$ (Fundamentalsatz d. Algebra)

$q_1 = q_2^* = \underline{q} = q_r + j q_i$

$\underset{\sim}{\Lambda} = \begin{pmatrix} \alpha + j\beta & 0 \\ 0 & \alpha - j\beta \end{pmatrix}$

$$\boxed{q = \begin{pmatrix} 1 \\ 2-j \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + j \begin{pmatrix} 0 \\ -1 \end{pmatrix}}$$

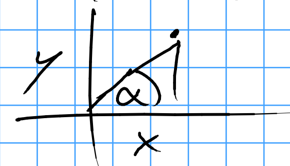
$\to$ reelle Normalform $\underset{\sim}{\Lambda}' = \begin{pmatrix} \alpha & -\beta \\ \beta & \alpha \end{pmatrix}$

$\underset{\sim}{Q}' = \begin{pmatrix} q_r & -q_i \end{pmatrix}$

entsprechend folgt:

$$\dot{\underline{\xi}}' = \Lambda' \underline{\xi}' \qquad \xi_{01} = |\xi_{01}| e^{j\varphi_0}$$

$$\underline{\xi}(t) = 2 e^{\alpha t} \begin{pmatrix} \mathrm{Re}\left(e^{j\beta t} \xi_{01}\right) \\ \mathrm{Im}\left(e^{j\beta t} \xi_{01}\right) \end{pmatrix} = \begin{pmatrix} \xi_1(t) \\ \xi_2(t) \end{pmatrix}$$



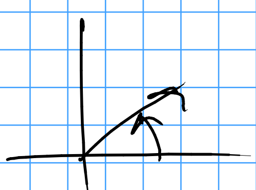
$\Rightarrow$ Polarkoordinaten:

$$\|\xi(t)\| = 2 e^{\alpha t} |\xi_{01}|$$

$$\arg(\underline{\xi}(t)) = \arctan\left(\frac{\xi_2(t)}{\xi_1(t)}\right) =$$

$$= \arctan\left(\frac{\mathrm{Im}\left(e^{j\beta t} |\xi_{01}| \cdot e^{j\varphi_0}\right)}{\mathrm{Re}\left(e^{j\beta t} |\xi_{01}| e^{j\varphi_0}\right)}\right) =$$

$$= \arctan\left(\frac{\mathrm{Im}\left(e^{j(\beta t + \varphi_0)} \cancel{|\xi_{01}|}\right)}{\mathrm{Re}\left(e^{j(\beta t + \varphi_0)} \cancel{|\xi_{01}|}\right)}\right)$$



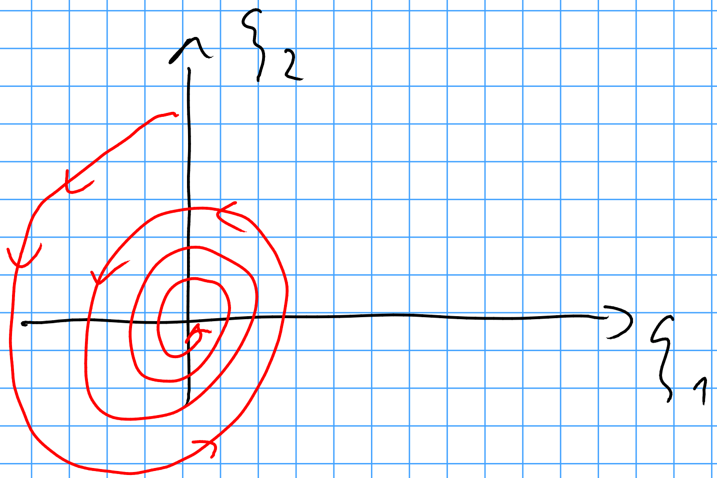
$$= \arctan\left(\frac{\sin(\beta t + \varphi_0)}{\cos(\beta t + \varphi_0)}\right) = \beta t + \varphi_0$$

$$\beta > 0$$

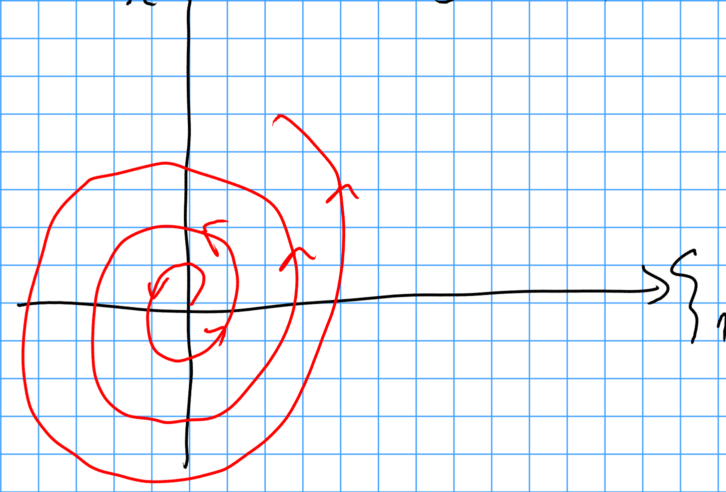
$\Rightarrow$ in der $\xi_1|\xi_2$-Ebene dreht sich der Strudel immer im Gegenuhrzeigersinn

Phasenportraits:

$\alpha < 0 \Rightarrow$ stabiler Strudel



$\alpha > 0$: instabiler Strudel



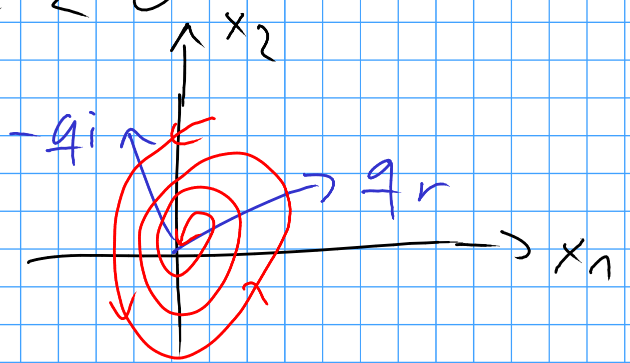
Übergang in x_1-x_2-Ebene:

$\underline{x} = \underline{Q}'\underline{\xi}'$ $\qquad \xi'_{e_1} = \begin{pmatrix}1\\0\end{pmatrix} \to q_r$

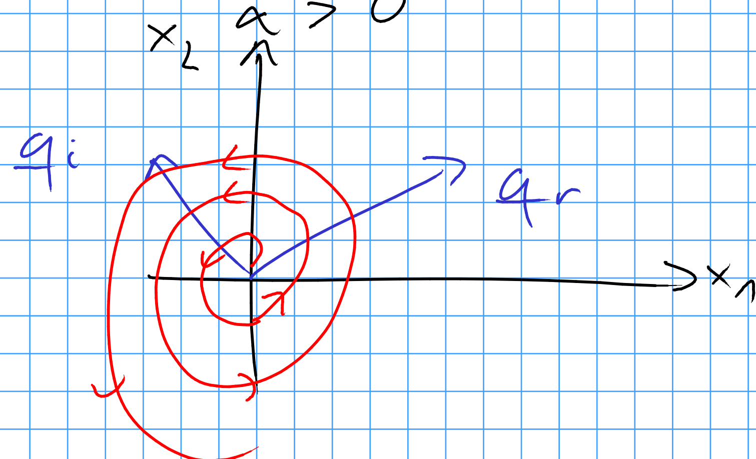
$\underline{Q}' = (q_r \;\; -q_i)$ $\qquad \xi'_{e_2} = \begin{pmatrix}0\\1\end{pmatrix} \to -q_i$

hier gilt nur mehr: immer von q_r zu $-q_i$

$\alpha < 0$



$\alpha > 0$



Einführung: Komplexe Wechselstromrechnung

→ Ausgangspunkt: eingeschwungener Zustand, sinusoidale Erregung

$$x(t) = A\cos(\omega t + \varphi_0)$$

→ Einführung eines komplexen Zeigers:

$$\underline{X} = A e^{j\varphi_0} = A\left(\cos(\varphi_0) + j\sin(\varphi_0)\right)$$

⇒ beinhaltet alle wesentlichen Informationen

→ Amplitude + Phasenlage

$$x(t) = \mathrm{Re}\left(\underline{X}\cdot e^{j\omega t}\right) = \mathrm{Re}\left(A e^{j\omega t + j\varphi_0}\right)$$

$$= A\cos(\omega t + \varphi_0)$$

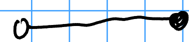
→ Weitere Motivation: Laplace/Fourier-Trafo

⇒ DGL → arithmet. Gleichung

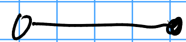
$$\frac{d}{dt}x(t) \circ\!\!-\!\!\bullet\ j\omega X(\omega)$$

Bauteilgleichungen:

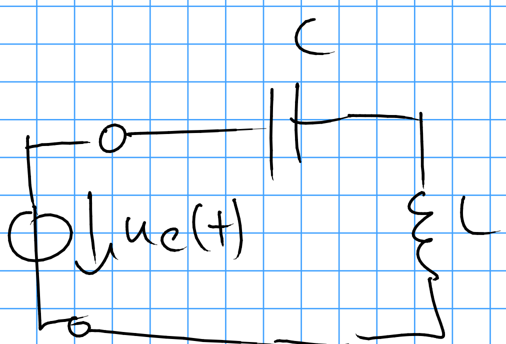
$i = G u$

$i_C = C \dot{u}_C$  $I_C = C j\omega U_C$ ← kompl. Leitwert → Admittanz

$= j\omega C \cdot U_C$

$u_L = L \dot{i}_L$  $U_L = j\omega L I_L$

$= j\omega L I_L$ ← kompl. Widerstand → Impedanz



$u_e(t) = \cos(\omega t + \varphi_0)$ $\underline{U}_e = e^{j\varphi_0}$

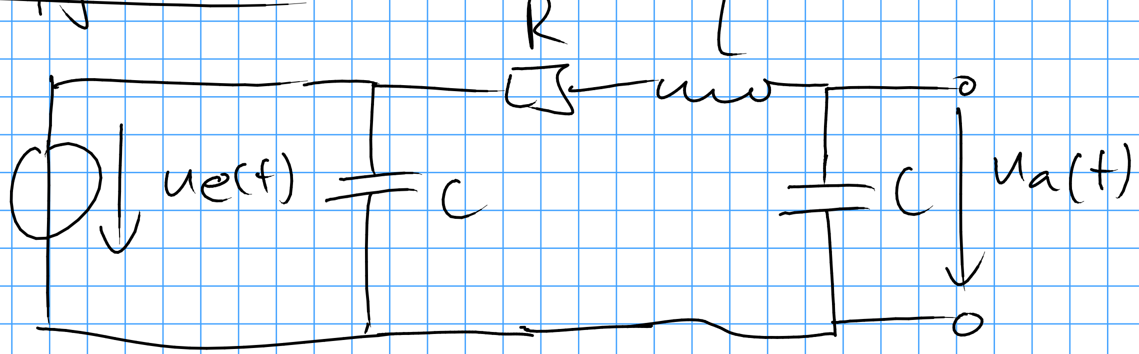
$$\underline{X} = j\omega L + \frac{1}{j\omega C}$$

Wirkwiderstand

$Z = R + jX$ ← Blindwiderstand

$|Z|$: Scheinwiderstand

Aufgabe 9:



a) $$\underline{X} = \frac{1}{j\omega C} \parallel \left(R + j\omega L + \frac{1}{j\omega C}\right) =$$

$$= \frac{\frac{1}{j\omega C}\cdot\left(R+j\omega L+\frac{1}{j\omega C}\right)}{\frac{1}{j\omega C}+R+j\omega L+\frac{1}{j\omega C}} =$$

$$= \frac{R+j\omega L+\frac{1}{j\omega C}}{2+j\omega RC-\omega^2 LC}$$

b) $u_e(t) = \hat{U}_e \cos(\omega t)$ $\quad \underline{U}_e = \hat{U}_e \cdot e^{j\varphi_0} = \hat{U}_e$

$$\underline{U}_a = \frac{\frac{1}{j\omega C}}{R+j\omega L+\frac{1}{j\omega C}}\cdot \underline{U}_e =$$

$$= \frac{1}{j\omega RC - \omega^2 LC + 1}\cdot\underline{U}_e$$

$$|\underline{U}_a| = |\underline{U}_e| \cdot \left| \frac{1}{j\omega RC - \omega^2 LC + 1} \right| =$$

$$= \hat{U}_e \cdot \frac{1}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$$

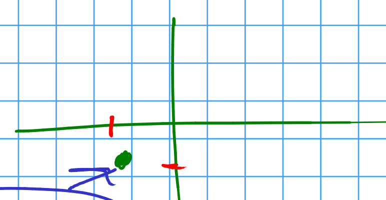
$$= 1V \cdot \frac{1}{\sqrt{(1-1\cdot 2\cdot 1)^2 + (1\cdot 1\cdot 1)^2}} = \frac{1V}{\sqrt{2}}$$

$$= 0{,}71\,V$$

$$\arg(\underline{U}_a) = \arg\left(\underline{U}_e \cdot \frac{1}{j\omega RC + 1 - \omega^2 LC}\right)$$

$$= \underbrace{\arg(\underline{U}_e)} + \arg\Bigg(\underbrace{\frac{1}{j\omega RC + 1 - \omega^2 LC}}_{\underline{U}}\Bigg)$$

$$= 0 + \arctan\left(\frac{\mathrm{Im}(\underline{U})}{\mathrm{Re}(\underline{U})}\right)$$



$$\boxed{\frac{1}{j\omega RC + 1 - \omega^2 LC} = \frac{(1-\omega^2 LC) - j\omega RC}{(1-\omega^2 LC)^2 + (\omega RC)^2}}$$

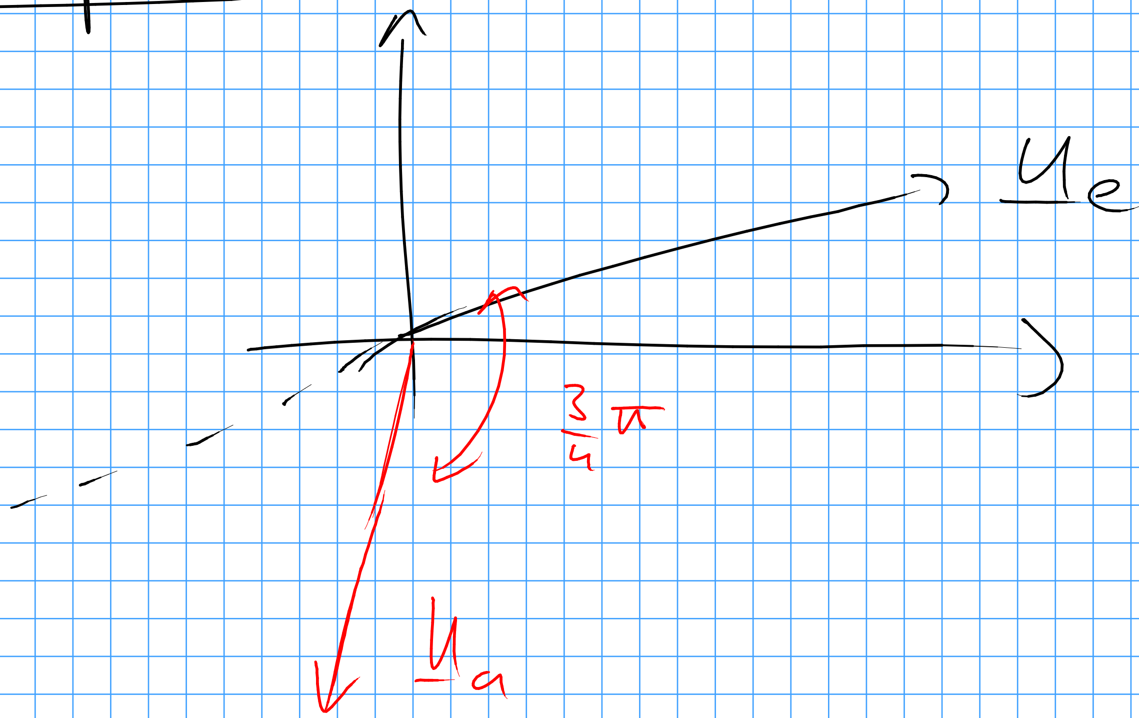
$$= \arctan\left(\frac{-\omega RC}{1-\omega^2 LC}\right) \;(\pm\pi)$$ ← Abhängig von der Lage des komplexen Punktes

$$= \arctan\left(\frac{-1}{-1}\right) - \pi$$ ← obiger Punkt liegt im dritten Quadranten!

$$= -\frac{3}{4}\pi$$

$$\underline{U}_a = 0{,}71\,V \cdot e^{j\varphi_1} = \underline{0{,}71\,V \cdot e^{-j\frac{3}{4}\pi}}$$

Interpretation:



⇒ Fortführung nächste Stunde