Stochastische Signale 17/12/2013

Zusatzaufgabe 1

$E[X] = 0 \qquad Var[X] = \sigma_x^2 = E[X^2] - \underbrace{E[X]^2}_{=0}$

$E[N] = \mu_N \qquad Var[N] = \sigma_n^2$

$Y = X + N$; X, N unkorreliert

$E[Y] = E[X+N] = E[X] + E[N] = \mu_N$

$Var[Y] = Var[X+N] = Var[X] + Var[N] = \sigma_x^2 + \sigma_n^2$

da X, N unkorreliert
$\Leftrightarrow Cov[X,N] = 0$

$\hat{X} = a(Y+b)$

a) $E[\hat{X}] = E[X] = 0 \Leftrightarrow E[aY + ab] = 0$

$a \cdot \mu_N + ab = 0 \quad | : (a \neq 0)$

$\mu_N = -b$

b) $Cov[R,Y] \stackrel{!}{=} 0 \qquad R = X - \hat{X}$

$Cov[R,Y] = E[RY] - \underbrace{E[R]}_{E[X-\hat{X}] = 0 - 0 = 0} E[Y] =$

$= E[RY]$

$$= E\left[(X - aY - ab)(X+N)\right] =$$

$$= E\left[X^2 + XN - aYX - aYN - abX - abN\right] =$$

$$= E\left[X^2\right] + \underbrace{E\left[XN\right]}_{E[X]\,E[N] = 0} - aE\left[(X+\underbrace{N)X}_{=0}\right] - aE\left[(X+N)N\right] - ab\mu_N$$

$$= \sigma_X^2 - aE\left[X^2\right] - \underbrace{aE\left[N^2\right] + a\mu_N^2}_{-a\,Var[N]} =$$

$$= \sigma_X^2 - a\sigma_X^2 - a\sigma_N^2 \overset{!}{=} 0$$

$$\Rightarrow a = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_N^2}$$

$\Rightarrow$ Orthogonalitätsprinzip: $E\left[(X-\hat{X})Y\right] = 0$

$\Leftrightarrow Cov[R, Y] = 0$

$\hookrightarrow$ Rekonstruktionsfehler ($X - \hat{X}$) steht senkrecht auf der Beobachtung.